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Fields Medal

"Nobel Prize" of mathematics

"[Alfred] Nobel had little esteem for mathematics. He was a practical man who ignored basic research. He never understood its importance and long-term consequences. But John Charles Fields did."

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The ***Fields Medal,*** officially known as ***International Medal for Outstanding Discoveries in Mathematics,*** is the highest scientific award for mathematicians not over forty years of age at each International Congress of the International Mathematical Union (therefore once every four years), since 1936 and regularly since 1950 at the initiative of the Canadian mathematician John Charles Fields. The purpose is to give recognition and support to young mathematical researchers having already made important contributions.

Introduction

How many winners of the Nobel Prize for mathematics can you name?

None, right? But cheer up. You can't think of one because there is no Nobel Prize for mathematics. The top award for mathematicians is the Fields Medal, which happens to bear the name of a Canadian.

In 1896 the Swedish inventor Alfred Nobel died rich and famous. His will provided the establishment of a prize fund. Starting in 1901 the annual interest was yearly awarded for the most important contributions to physics, chemistry, physiology or medicine, literature, and peace. The economics prize appeared later. The Central Bank of Sweden founded it in 1968 to commemorate its 300th anniversary.

Why did Nobel choose these fields?

The inventor of dynamite loved chemistry and physics. Literature was his great passion; in spite of a busy life, he found time to read and write fiction. Medicine and peace were natural choices for the benefit of humankind. But what about mathematics?

Whatever the reason, Nobel had little esteem for mathematics. He was a practical man who ignored basic research. He never understood its importance and long-term consequences.

Condition for future prizewinners

The Fields Medal Committee is chosen by the Executive Committee of the International Mathematical Union and is normally chaired by the IMU President. It is asked to choose at least two, with a strong preference for four, Fields Medallists, and to

have regard in its choice to representing a diversity of mathematical fields. A candidate's 40th birthday must not occur before January 1st of the year of the Congress at which the Fields Medals are awarded.

The name of the Chair of the Committee is made public, but the names of other members of the Committee remain anonymous until the award of the prize at the Congress. If a former student (Ph.D. thesis only) of a Committee member is seriously considered, such a member shall not continue to serve on the Committee for is final decision.

History of the Fields Medal

The history of the Fields Medal begins in the Committee of the International Congress set up by the University of Toronto in November of 1923.

Fields was its chairman, and his colleague J.L.Synge the secretary. The first recorded mention of the medal is in a meeting of that committee on February 24, 1931 where it is "resolved that the sum of $2,500 should be set apart for two medals to be awarded in connection with successive International Mathematical Congresses (IMC) through an international committee appointed for such purpose initially by the executive of the IMC".

At the next meeting, in January 1932, Fields indicated that the idea of the medal had the support of the major mathematical societies of France, Germany, Italy, Switzerland and the United States, and he also outlined the principles behind the proposed medal. The genesis of the rule that it be awarded only to mathematicians no older than forty is evidently the statement that "… while it was in recognition of work already done, it was at the same time intended to be an encouragement for further achievement on the part of the recipients and a stimulus to renewed effort on the part of others". Then he continued "The Committee might ease matters by saying that they had decided to make the awards along certain lines not alone because of the outstanding character of the achievement but also with a view to encouraging further development along these lines".

Of course, in spite of Fields's intentions, the medal became known as the Fields Medal when it was awarded for the first time in Oslo in 1936. The monetary value of the Fields Prize is presently $15,000Can (about $9500US), hardly compare with its stature as the "Nobel Prize in Mathematics".

Why Nobel chose not to establish a prize in mathematics?

It is not known why Nobel chose not to establish a prize in Mathematics, although there are several theories about the lack of one. The persistent rumor that Nobel did not establish a prize in mathematics because Mittag-Leffler had an affair with Nobel's wife is certainly incorrect. Nobel

never married. But the other version of this rumor, founded on hostility between Nobel and Mittag-Leffler, may be correct though there is no documentation to support it.

There is no doubt that Nobel and Mittag-Leffler knew each other. Mittag-Leffler was one of the most prominent Swedish scientists at the time. In 1890 Nobel turned down Mittag-Leffler's proposal to fund a professorship for Sonya Kovalevsky at the Stockholms Högskola where Mittag-Leffler was a professor. The Högskola was named as a beneficiary in Nobel's first will (1883), but not in his final will (1896). According to, the rector of the Högskola, Otto Pettersson, and Svante Arrhenius, let it be known "that Nobel's dislike for Mittag-Leffler had brought about what Pettersson called the `Nobel flop' " (the term he used to describe the dropping of the Högskola from Nobel's will).

One wonders whether the hostility between Nobel and Mittag-Leffler, and the friendship between Fields and Mittag-Leffler, were factors in Fields's establishment of his award. Ironically, Mittag-Leffler (as well as Arrhenius) was, in the first few years after Nobel's death in 1896, of "decisive importance … in shaping the decisions and hence the international standing of the [Nobel] prizes".

After Fields death

Fields then proceeded with the planning of the award of the first medals, but fell ill in May of 1932 and died 3 months later. Before his death, with Synge at his bedside, he made his will. It included an amount of $47,000 to be added to the funds for the medal. Synge carried Fields's proposal to the Congress in Zürich in September of that year. One month later the ICM adopted the proposal with an overwhelming majority of votes. To respect Fields's wishes, the award was named the International Medal for Outstanding Discoveries in Mathematics, but everybody called it the Fields Medal. A committee consisting of G.D.Birkhoff, Carathéodory, E.Cartan, Severi and Takagi was formed to make the first awards at the Oslo Congress in 1936. They chose Lars Ahlfors of Finland and Jesse Douglas of the U.S.A.

Unfortunately, war again intervened, and the next IMC was not held until 1950, in Cambridge, when Laurent Schwartz and Atle Selberg were selected as the Fields Medallists.

John Charles Fields (1863-1932)

Fields was a pioneer and visionary who recognized the scientific, educational, and economic value of research in the mathematical sciences. Fields spent many of his early years in Berlin and in Paris and Göttingen (Germany), the principal

mathematical centres of Europe of that time. These experiences led him, after his return to Canada, to work for the public support of university research, which he did very successfully. He also organized and presided over the 1924 meeting of the International Congress of Mathematicians in Toronto.

There is no Nobel Prize in mathematics, and Fields felt strongly that there should be a comparable award to recognize the most outstanding current research in mathematics. With this in mind, he established the International Medal for Outstanding Discoveries in Mathematics, which, contrary to his personal directive, is now known as the Fields Medal.

Bibliography

John Charles Fields was born in Hamilton, Ontario, then Upper Canada, in 1863. Fields graduated from the University of Toronto in 1884, and then left to study at Johns Hopkins University. Fields was awarded a Ph.D. in 1887. His thesis was entitled *Symbolic Finite Solutions and Solutions by Definite Integrals of the Equation* $d^n y/dx^n = x^m y$, and was published in the American Journal of Mathematics in 1886. After teaching at Johns Hopkins for two years, he joined the faculty of Allegheny College in Meadville, Pennsylvania. Fields was understandably dissatisfied with the state of mathematics in North America at that time, and in 1891 he left for Europe to spend the next 10 years there.

Fields's years in Europe, mainly in Berlin but also in Göttingen and Paris, influenced him deeply and reinforced his convictions about the importance of mathematical research. He mingled with many of the greatest mathematicians of that time - Karl Weierstrass, Felix Klein, Ferdinand Georg Frobenius, Max Planck, Fuchs, Hensel– and changed his mathematical interests to algebraic functions in which he published many papers during the rest of his mathematical career. He also developed there a life-long friendship with the Swedish mathematician Gösta Mittag-Leffler. Fields returned to Canada in 1902 as a special lecturer at the University of Toronto. He remained at the University of Toronto for the rest of his life, and became a Fellow of the Royal Society of Canada in 1909 and of the Royal Society of London in 1913.

After completing the proceedings of the 1928 Congress, he proceeded with the planning of the award of the first medals, but fell ill in May of 1932 and died in August.

Fields is buried in the Hamilton Cemetery overlooking the western end of Lake Ontario ("Cootes Paradise", where McMaster University is also located).

Field's gravestone. It is set into the ground flat, is about 22 inches by 16 inches and simply says, "J.C.Fields, born May 14, 1863, died August 9, 1932 ".

Field's letter

This is the original letter by Fields creating. It has been written during the few months before his death. Notice that no mention is made about the 40 year-old limit, and that the medal shouldn't bear anybody's name.

It is proposed to found two gold medals to be awarded at successive International Mathematical Congress for outstanding achievements in mathematics. *Because of the multiplicity of the branches of mathematics and taking into account the fact that the interval between such congresses is four years it is felt that at least two medals should be available. The awards would be open to the whole world and would be made by an International Committee.*

The fund for the founding of the medals is constituted by balance left over after financing the Toronto congress held in 1924. This must be held in trust by the Government or by some body authorized by government to hold and invest such funds. It would seem that a dignified method for handling the matter and one which in this changing world should most nearly secure permanency would be for the Canadian Government to take over the fund and appoint as his custodian say the Prime Minister of the Dominion or the Prime Minister in association with the Minister of Finance. The medals would be struck at the Mint in Ottawa and the duty of the custodian would be simply to hand over the medals at the proper time to the accredited International Committee.

As things are at present a practical course of procedure would seem to be for the Executive Committee of a Congress to appoint a small international committee authorized to add to its number and call into consultation other mathematicians as it might deem expedient. The Committee would be expected to decide on the ones to whom the awards should be made thirty months in advance of the following Congress. Its decisions would be communicated to the President and Secretary of the Organizing Committee of the Congress, this Committee having the duty of communicating to the Prime Minister of Canada the names of the recipients in order that the medal might be prepared in time and forwarded to the president of the Organizing Committee. Immediately on the appointment of the Executive Committee of the Congress the medals would be handed over to its President. The presentation of the medals would constitute a special feature at some general meeting of the Congress.

In the above arrangements the role of the Organizing Committee might be taken over by the Executive of the International Mathematical Union at some time in the future when that organization has been generally accepted.

In coming to its decision the hands of the IC should be left as free as possible. It would be understood, however, that in making the awards while it was in recognition of work already done it was at the same time intended to be an encouragement for further achievement on the part of the recipients and a stimulus to renewed effort on the part of others.

In commenting on the work of the medalists it might be well to be conservative in one's statements to avoid envidious comparisons explicit or implied. The Committee might ease matters by saying they have decided to make the awards along certain lines not alone because of the outstanding character of the achievement but also with a view to encouraging further development along these lines. In this connection the Committee might say that they had elected to select subjects in Analysis, in Geometry, in the Theory of Groups, in the Theory of Numbers etc. as the case might be. When the Committee had come to an agreement in this sense the claims for recognition of work done along the special lines in question could be considered in detail by two smaller groups or subcommittees with specialized qualifications who would have authority to take into consultation or add to the subcommittees other mathematicians of specialized knowledge.

With regard to the medals themselves, I might say that they should each contain at least 200 dollars worth of gold and be of a fair size, probably 7.5 centimeters in diameter. Because of the international character the language to be employed it would seem should be Latin or Greek? The design has still to be definitely determined. It will have to be decided on by artists

***in consultation with mathematicians**. The suggestions made in the preceding are tentative and open to consideration on the part of mathematicians.*

It is not contemplated to make an award until 1936 at the Congress following that at Zurich during which an international Medal Committee should be named.

The above programme means a new departure in the matter of international scientific cooperation and is likely to be the precursor of moves along like lines in other sciences than mathematics.

***One would hear again emphasized the fact that the medals should be of a character as purely international and impersonal as possible. There should not be attached to them in any way the name of any country, institution or person**.*

Perhaps provision could be made as soon as possible after the appointment of the Executive of the Zurich Congress for the consideration by it of the subject of the medals, and the appointment without undue delay of a Committee and the awards of the medals to be made in connection with the Congress of 1936.

Suggestions with regard to the design of the medals will be welcome.

(Signed) J.C. Fields Research Professor of Mathematics University of Toronto

The Fields Medal – description

The medal, struck by the Royal Canadian Mint, is a gold-plated cast, 25 centimetres in diameter. Designed in 1932 by the Canadian sculptor Robert Tait McKenzie.

McKenzie had his own impressions of the greatest mathematician of antiquity. In 1932 he wrote to Synge: “I feel a certain amount of complacency in having at last given to the mathematical world a version of Archimedes that is not decrepit, bald-headed and myopic, but which has the fine presence and assured bearing of the man who defied the power of Rome.”

Obverse

The head represents Archimedes facing right. (1)In the field is the word **ΑΡΧΙΜΗΔΟΥΣ** in Greek capitals and (2)the artist's monogram and date **RTM, MCNXXXIII**. (3) The inscription reads: **TRANSIRE SUUM PECTUS MUNDOQUE POTIRI**.

The inscriptions mean (1) "of Archimedes", namely the face of Archimedes. (2) R(obert) T(ait) M(cKenzie), that is the name of the Canadian sculptor who designed the medal. The correct date would read: "MCMXXXIII" or 1933. The second letter M has to be substituted for the false N. (3) The Latin inscription from the Roman poet Manilius surrounding his image may be translated “to pass beyond your understanding and make yourself master of the universe”. The phrase comes from

Manilius's *Astronomica* 4.392 from the first century A.D. The complete passage is *The object of your quest is God; you are seeking to scale the skies and though born beneath the rule of fate, to gain knowledge of that fate; you are seeking to pass beyond your understanding and make yourself master of the universe.*

Reverse

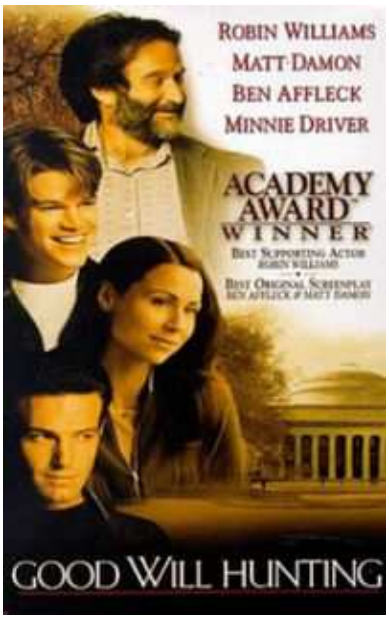
The inscription on the tablet reads:

CONGREGATI
EX TOTO ORBE
MATHEMATICI
OB SCRIPTA
INSIGNIA
TRIBUERE

It means:"The mathematicians having congregated from the whole world awarded (this medal) because of outstanding writings". The verb form "tribuere" (the first "e" is a long vowel) is a short form of "tribuerunt". Behind the inscription is a laurel branch, and a diagram of a sphere contained in a cylinder from an engraving thought to have been on Archimedes' tomb.

The Fields Medal in popular culture

In the 1997 film *Good Will Hunting*, fictional MIT (Massachusetts Institute of Technology) professor Gerald Lambeau (played by Stellan Skarsgård) is described as having been awarded a Fields Medal for his work in combinatorial mathematics.



Fields Medal Prizewinners

Since 1936, 44 mathematicians have received the Fields Medal. The first medals of the 21st century were awarded in the year 2002 in China.

- 2002: **Laurent Lafforgue** *(France)* , **Vladimir Voevodsky** *(Russia/US)*
- 1998: **Richard Ewen Borcherds** *(GB),* **William Timothy Gowers** *(GB),* **Maxim Kontsevich** *(Russia),* **Curtis T. McMullen** *(US),* and a silver plate was given to **Andrew Wiles** *(GB)*
- 1994: **Efim Isakovich Zelmanov** *(Russia;39 years)*, **Pierre-Louis Lions** *(France;38),* **Jean Bourgain** *(Belgium;40),* **Jean-Christophe Yoccoz** *(France;36)*
- 1990: **Vladimir Drinfeld** *(USSR;36),* **Vaughan Frederick Randal Jones** *(New Zealand;38),* **Shigefumi Mori** *(Japan;39),* **Edward Witten** *(US;38)*
- 1986: **Simon Donaldson** *(GB;27),* **Gerd Faltings** *(West Germany;32),* **Michael Freedman** *(US;35)*
- 1982: **Alain Connes** *(France;35)* , **William Thurston** *(US;35),* **Shing-Tung Yau** *(China;33)*
- 1978: **Pierre Deligne** *(Belgium;33),* **Charles Fefferman** *(US;29),* **Grigory Margulis** *(USSR;32),* **Daniel Quillen** *(US;38)*
- 1974: **Enrico Bombieri** *(Italy;33),* **David Mumford** *(US;37)*
- 1970: **Alan Baker** *(GB;31),* **Heisuke Hironaka** *(Japan;39),* **Sergei Petrovich Novikov** *(USSR;32),* **John Griggs Thompson** *(GB;37)*
- 1966: **Michael Atiyah** *(GB;37),* **Paul Joseph Cohen** *(US;32),* **Alexander Grothendieck** *(France;38),* **Stephen Smale** *(US;36)*
- 1962: **Lars Hörmander** *(Sweden;31),* **John Milnor** *(US;31)*
- 1958: **Klaus Roth** *(GB;32),* **Rene Thom** *(France;35)*
- 1954: **Kunihiko Kodaira** *(Japan;39),* **Jean-Pierre Serre** *(France;27)*
- 1950: **Laurent Schwartz** *(France;35),* **Atle Selberg** *(Norway;33)*
- 1936: **Lars Ahlfors** *(Finland, 29),* **Jesse Douglas** *(US;39)*

Fields Medal Winners 2002

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