# People Maths: Active Learning of Mathematics without a textbook

## Material for workshop

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This is an abstract of a number of items from a workshop session

We first set out Learning Objectives of Session:
**"By the end of the session you should:**
1. Be able to see approaches to make use of resources, (including human resources);
2. Be able to see approaches for the benefit of learners of mathematics of a variety of learning styles
3. Have some ideas to explore afterwards, using pencil and paper methods"

A rule for learning was shared :
"There is more than one teacher in this classroom" - to be interpreted as implying that people could and should learn form each other, that this was a collaborative learning experience; i.e. that the "official" teacher was not the only one from whom learners could gain learning benefits.

**"People Maths"** as a type of activity was defined:
"People Maths" uses people to form the moving pieces of a mathematical activity, be it a puzzle, a sum, a diagram or a demonstration. This form of learning particularly benefits those who are active learners, kinaesthetic or visual learning style. There are also benefits for making learners use precise mathematical language, and discuss proofs; these are also benefits for those, like the author, for whom English is an Additional Language.

What is important is that people act out these situations, and do not turn, individually, to diagrams and paper and pencil calculations, too quickly. The mathematical and social benefits from the interactions between the participants must not be underestimated.

**A)** An activity called **Square Pairs**, invented by the author, was introduced, after everyone had been given a number from 1 up to 30.

An even number of people are given numbers from 1 to N. They are then asked to arrange themselves into pairs so that the SUM of the numbers of EVERY PAIR of people is a SQUARE number. For a successful game EVERY person must be in such a pair. People are guided to use "trial and improvement" to secure such success.

*Questions to consider:*
*Which starting even numbers is it possible to (fully) square pair?*
*For which it is it impossible? For which is it possible in more than one way?*

*What patterns or generalisations are there for "square pairing" numbers in the <u>simplest</u> way? What proofs can be given for such processes?*

*Which numbers can be square paired by pairing 1 to N, 2 to N-1, 3 to N-2, and so on? What can be said about such numbers, mathematically?*

**B) ROBAPS**
Reverse Order By Adjacent Pair Swaps

A group of 6 people are given numbers 1 to 6 and stand in order in one line, numbers visible to all. They can swap in adjacent pairs. The number of swaps until the initial order has been reversed is counted.

*Questions to consider:*
*What is the number of swaps required to reverse, via the ROBAPS process, a line of 100 people? Is there a "best" process?*

**C) Equal Sums**

Three people stand at the corner/vertex of a triangle. Three others also stand, each one halfway between a pair at a triangle vertex. They each have one of the numbers 1-6.

*Questions to consider:*
*How can they arrange themselves so that the 3 numbers along the sides of the triangle give the same total? How many ways are such equal sums possible?*

*What if there are 9 people numbered 1-9, one at each corner, two people between them?*

**D) Back Front Shuffles**
6 people each with a number 1-6 stand in a line in numerical order [1 2 3 4 5 6]
A new row in front of them is made, moving in turn. Each completed row counts as one round.
The person at the back starts a new line, then the person at the front moves, then the next from the back, then the next from the front : result [6 1 5 2 4 3]
Count the number of rounds until you reach the original line-up sequence.

*Questions to consider:*
*What do you think will happen for other numbers of people? Try for 5, 7, 8, 10.*

*Can you predict how many ronds will be needed for N people?*

**E) Infinite Sums**

1. A person stands at one end of a room. She walks halfway to the other side. She then walks half the distance that remains, then again half the distance that remains, and so on. This illustrates that $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots =$ (what) ?

2. Two people start at opposite ends of a room facing each other. Each moves, first, $\frac{1}{3}$ of the way towards the other; then again $\frac{1}{3}$ of what remaining space, and so on.
Between the two of them they meet – where is this meeting point?
What infinite sum does this illustrate?

*Questions to consider:*
*What other infinite sums could be acted out in this way, and how?*

**F) Pair Product Sums**
A group of 8 people, numbered 1-8 line up in a random pairing.
The manager calls out the number pairs, and the recorder writes down the numbers. The product of each pair is calculated, and the total of these 4 products is added. Other pairings should then be tried, by the manager's or team's choice.

*Questions to consider:*
*Problem 1: How should the manager arrange the team so that the pair product sum is as large as possible? Is there a logical reason why this arrangement gives the maximum? What is this maximum?*
*Problem 2: How should the manager arrange the team so that the pair product sum is as low as possible? Why might this arrangement be best? What is this minimum?*

**G) Frogs and Tadpoles**
7 chairs are arranged in a row. 3 boys sit at one end, 3 girls at the other, with an empty chair in the middle. The problem is to rearrange the group so that the boys and girls swap sides. The only moves allowed are to slide one space into an empty chair; or to jump over one other person into an empty chair.

*Questions to consider:*
*How many moves are needed for this rearrangement? What if there are 2 boys and 2 girls, with one chair between them? Can a general formula be found for N boys, N girls, with one chair between them?*
*What if there are N boys, M girls, with one chair between them?*
*What if there are N boys, M girls, and C chairs between them?*

[This is a game originally called Frogs, where usually if the process is stuck, with no legal moves available, the game has to restart. In what developed as a computer-based pair of games using this idea, a version called Tadpoles allows the reversal of moves, but the total number of moves is still counted. This gives a hint that perhaps there is a solution with fewer moves, and so is a better version as a People Maths game.]

**H) Crossing the Bridge**

A string bridge across a ravine can carry up to 2 people. When 2 people walk across the bridge they can only go at the speed of the slowest person. They are crossing at night, a lamp lighting their way.There is a family of 4 who wish to cross. They take 1, 2, 5, and 10 minutes individually. They have just one lamp, which must be used on each crossing.

*Questions to consider:*
*Can they get across in less than 18 minutes? What is the strategy to minimise this time?*
*Is there more than one solution?*

*Can you invent other such crossing puzzles?*
[A famous similar puzzle also nice to carry out as a People Maths activity has a woman and her two children needing to cross a river. The boat can only carry one adult, or two children. How do they get across? Variations to explore, then, are other numbers of adults and children]

Read the book!
**People Maths Hidden Depths by Alan Bloomfield and Bob Vertes**
**Published by ATM (Association of Teachers of Mathematics) 2005**
**ISBN 1898611378  www.atm.org.uk (£15).**
**Book 2 due out April 2008!**

An activity also explored, as a way of doing mathematics without a textbook, was
**Mathematical Origami and proof**
1. Show how to take a piece of A4 and fold a square – Prove it is a square. Y*ou will need to fold a 45° angle..*
Show how to make mathematical good use of the remnants. *4 of them, if superimposed with a common centre, will make a regular octagon.*
2. Show how to fold a sheet of A4 most simply into a kite (2 folds) – and prove it is a kite. *You will need to know the paper sizes of A4 give side lengths in ratio √2 : 1.*
3. Show how to trisect a length of paper using trial and improvement – proof by demonstration.
4. Show how to fold a 60° angle. Prove it's a 60° angle. Can you use what you have done also fold a regular hexagon? An equilateral triangle?
5. Prove <u>by paper-folding</u> (no tearing/cutting allowed) that the sum of the angles in a triangle is 180°. *Hint: Start by folding the top corner down to meet the base line*
6. Show to fold a regular pentagon most efficiently. *Hint: Start by folding the sheet so that the two diagonally opposite corners meet.*