People Maths: Active Learning of Mathematics without a textbook

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Mathematics at school today and tomorrow

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Learning Objectives of Session

By the end of the session you should:

- Be able to see approaches to make use of resources, (including human resources);
- Be able to see approaches for the benefit of learners of mathematics of a variety of learning styles
- Have some ideas to explore afterwards, using pencil and paper methods

A rule for learning

There is more than one teacher in this classroom

People Maths

- “People Maths” uses people to form the moving pieces of a mathematical activity, be it a puzzle, a sum, a diagram or a demonstration.
- This form of learning particularly benefits those who are active learners, kinaesthetic or visual learning style.
- There are also benefits for making learners use precise mathematical language, and discuss proofs

Square Pairs

- An even number of people are given numbers from 1 to N
- They are then asked to arrange themselves into pairs so that the SUM of the numbers of EVERY PAIR of people is a SQUARE number.
- For a successful game EVERY person must be in such a pair.
- Use "trial and improvement" to secure such success.

Questions to consider:

- *Which starting even numbers is it possible to (fully) square pair?*
- *For which it is it impossible? For which is it possible in more than one way?*
- *What patterns or generalisations are there for "square pairing" numbers in the simplest way? Proofs?*

ROBAPS

- Reverse
- Order
- By
- Adjacent
- Pair
- Swaps

- A group of 6 people are given numbers 1 to 6 and stand in order in one line, numbers visible to all
- They can swap in adjacent pairs. The number of swaps until the initial order has been reversed is counted.
- What is the number of swaps required to reverse, via the ROBAPS process, a line of 100 people?

Equal Sums

- Three people stand at the corner/vertex of a triangle
- Three others stand, each one halfway between a pair at a triangle vertex
- They each have one of the numbers 1-6
- How can they arrange themselves so that the 3 numbers along the sides of the triangle give the same total?
- How many ways are such equal sums possible?

- What if there are 9 people numbered 1-9, one at each corner, two people between them?

Back Front Shuffles

- 6 people each with a number 1-6 stand in a line in numerical order [1 2 3 4 5 6]
- A new row in front of them is made, moving in turn. Each completed row counts as one round.
- The person at the back starts a new line, then the person at the front moves, then the next from the back, then the next from the front : result [6 1 5 2 4 3]
- Count the number of rounds until you reach the original line-up sequence.
- What do you think will happen for other numbers of people? Try for 5, 7, 8, 10

Back to Front Shuffle

Original order	1	2	3	4	5	6
After 1 round	6	1	5	2	4	3

Infinite Sums

- 1. A person stands at one end of a room. She walks halfway to the other side. She then walks half the distance that remains, then again half the distance that remains, and so on.
- $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots = ?$

- 2. Two people start at opposite ends of a room facing each other. Each moves, first $\frac{1}{3}$ of the way towards the other; then again $\frac{1}{3}$ of what remaining space, and so on.
- Between the two of them they meet –where?
- What infinite sum does this illustrate?

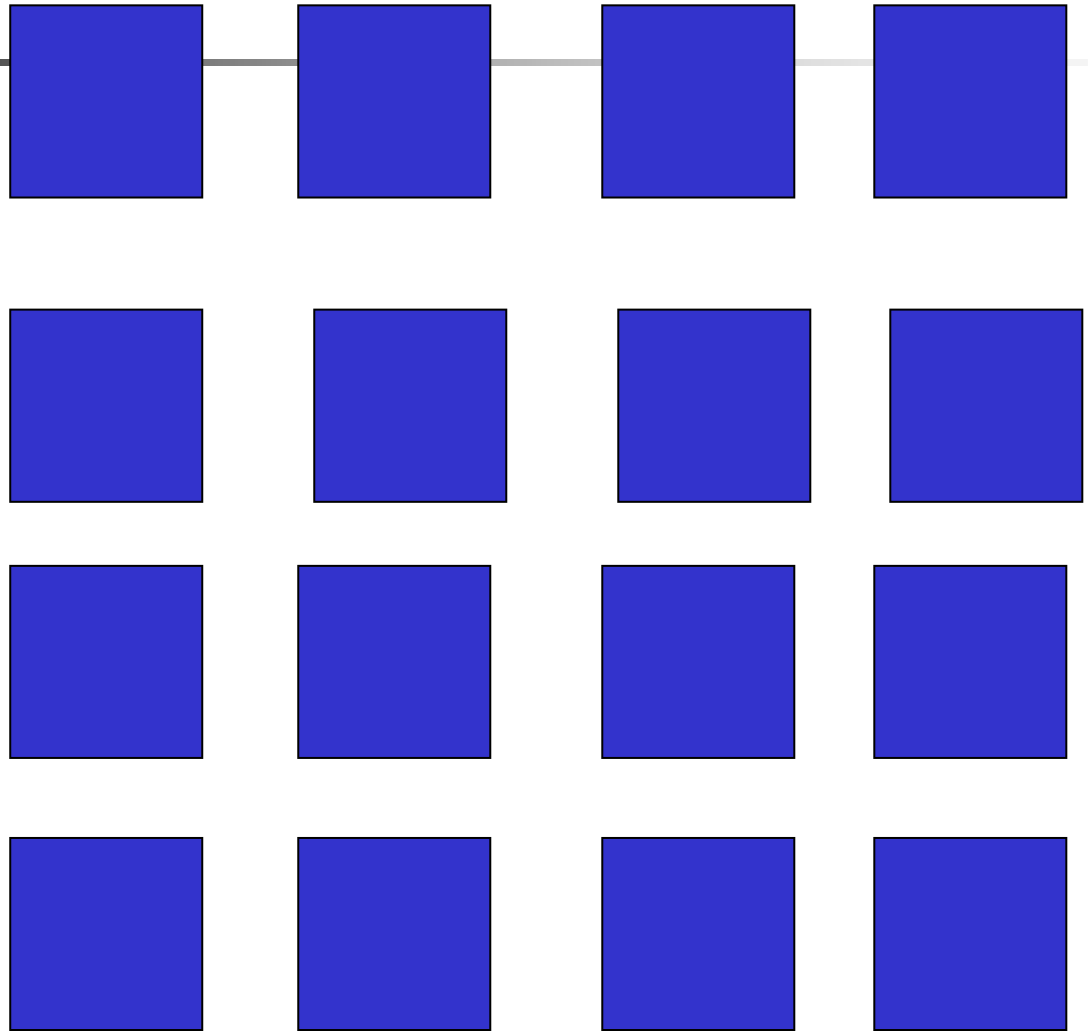
Pair Product Sums

- A group of 8 people, numbered 1-8 line up in a random pairing.
- The manager calls out the number pairs, and the recorder writes down the numbers. The product of each pair is calculated, and the total of these 4 products is added. Other pairings should then be tried, by the manager's or team's choice.
- Problem 1: How should the manager arrange the team so that the *pair product sum* is as large as possible? Is there a logical reason why this arrangement gives the maximum? What is this maximum?
- Problem 2: How should the manager arrange the team so that the *pair product sum* is as low as possible? Why might this arrangement be best? What is this minimum?

Frogs and Tadpoles

- 7 chairs are arranged in a row
- 3 boys sit at one end, 3 girls at the other, with an empty chair in the middle.
- The problem is to rearrange the group so that the boys and girls swap sides.
- The only moves allowed are to slide one space into an empty chair; or to jump over one other person into an empty chair.
- How many moves are needed?

Grab a Chair



Mini-Sudoku (a)

1			
			4
	2		
		3	

Mini-Sudoku (b)

1			
			4
2			
		3	

Read the book!

- People Maths Hidden Depths
- By Alan Bloomfield and Bob Vertes
- Published by ATM 2005
- ISBN 1 898611 37 8
- Association of Teachers of Mathematics www.atm.org.uk (£15).
- Book 2 due out April 2008!

Mathematical Origami and proof

- 1. Show how to take a piece of A4 and fold a square – Prove it is a square (you will need to fold a 45° angle).

  Show how to make mathematical good use of the remnants.
- 2. Show how to fold a sheet of A4 most simply into a kite (2 folds) – and try to prove it is a kite.
- 3. Show how to trisect a length of paper using trial and improvement – proof by demonstration.
- 4. Show how to fold a 60° angle. Prove it's a 60° angle. Can you fold a regular hexagon? An equilateral triangle?
- 5. Prove that the sum of the angles in a triangle is 180°.
- 6. Show to fold a regular pentagon most efficiently.

Crossing the Bridge 1

- A string bridge across a ravine can carry up to 2 people.
- When 2 people walk across the bridge they can only go at the speed of the slowest person.
- They are crossing at night, a lamp lighting their way.
- There is a family of 4 who wish to cross.
- They take 1, 2, 5, and 10 minutes individually.
- They have just one lamp, which must be used on each crossing.
- Can they get across in less than 18 minutes?