First steps in cooperative learning

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Abstract *The authors want to present a method of cooperative learning to promote understanding in mathematical problem solving. Cooperative learning is an alternative way of teaching. The Hungarian teachers of Grammar Schools rarely apply this method in mathematical problem solving. We made videotaped teaching experiments in three Grammar Schools with different grades of students (9-11). For our investigations we chose an applied extreme value problem, which the students could solve at different levels of knowledge. It was rich in solving methods, in applying different and various strategies. In our opinion the different solutions for the same problem should get an important place, it is a very good possibility to demonstrate the connections between different mathematical topics. We investigated how the students could find more solutions of the posed problem by cooperative learning. We made four hypotheses: successfulness, problem solving strategies, sex and free choice of small groups. The completion of the experiment confirmed some statements of didactical literature.*
Keywords: Problem solving, teaching-learning processes, cooperative learning
MESC: C30, C70

Method of the cooperative learning

Cooperative learning is the instructional use of small groups. Students work together to maximize their own and each other's learning. Class members are organized into small groups after receiving instructions from the teacher.

Use of the cooperative learning

1. Students learn knowledge, skills, strategies or procedures in a cooperative group.
2. Students apply the knowledge or perform the skill, strategy or procedure alone to demonstrate their personal mastery of material.
3. Students learn together and then perform alone.

Teaching experiments

The following problem was posed to the students:

A farmer has an adjustable electric fence that is 120 m long. He uses this fence to enclose a rectangular grazing area. Find the maximum area he can close.

This problem was an applied mathematical problem. We can solve it at different levels of the knowledge. It was rich in solving methods, in applying different and various strategies. In our opinion the different solutions for the same problem should get an important place, it is a very good possibility to demonstrate the connections between different mathematical topics. We investigated how could the students find the solutions of this extremal problem in form of cooperative learning.

In our case we choose the method of cooperative learning, learning in small groups (2-6 students). This was the first occasion to try and apply the method of cooperative learning for our mathematics teachers. There were different circumstances at different schools.

The teacher was a man in the high school Fazekas, and he taught the whole class. In this class the groups were bigger ones during this experiment (4-5 persons).

In the high school Református the teacher was a woman. In their groups were 4, respectively 2-6 persons.

The male teacher of the high school Kossuth taught the whole class which has math speciality. The groups was formed on the decision of students, as they wanted and could collaborate and as they wanted to share the roles in the groups.

Our hypotheses were:

- if we pose for the small group of students the goal to solve the extremal problem they will be successful in finding a lot of different solutions
- the strategies and contents of the solving process depends on the age of the students
- the result does not depend from the sex of teachers or students
- the free choose of the small groups leads to a various kind of mixed groups.

Some remarks about the Hungarian curriculum (grades 9-11) in connection with our problem:

grade 9. students could learn mathematics only 3 hours per week (knowledge of quadratic functions, some transformations of quadratic functions, squaring a quadratic form)

grade 10. students could learn mathematics only 3 hours per week (solving quadratic equations and they know the connection of arithmetic and geometrical means)

grade 11. it is possible to learn mathematics both 3 and 3+2 hours per week (solving extremal problems with the help of derivation)

The expected solutions of the extremal problem

We previously showed the teachers 7 different solutions of the given problem but we expected to get about 5 solutions from the students.

Our expected solutions were

1. Systematical trials
2. Using characteristic of the quadratic functions a) algebraic methods b) graphical methods
3. Geometrical methods (modelling)
4. Method of derivation
5. Non standard methods

Some details from the students' work

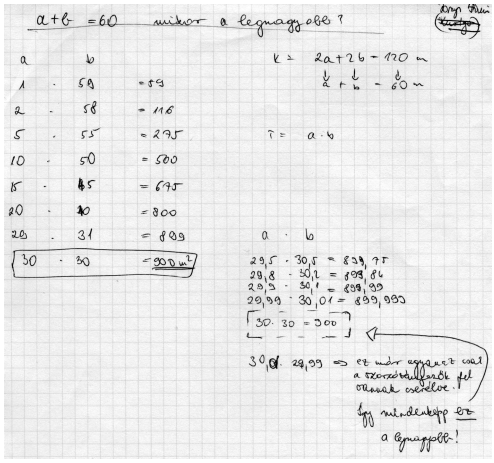

Figure 1. - Systematical trial in Gabi's notebook

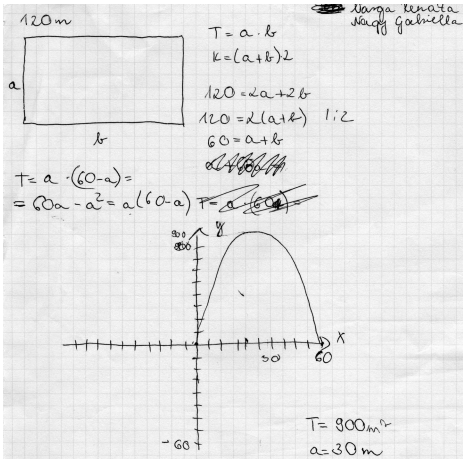

Figure 2. - Quadratic functions and Graphical method

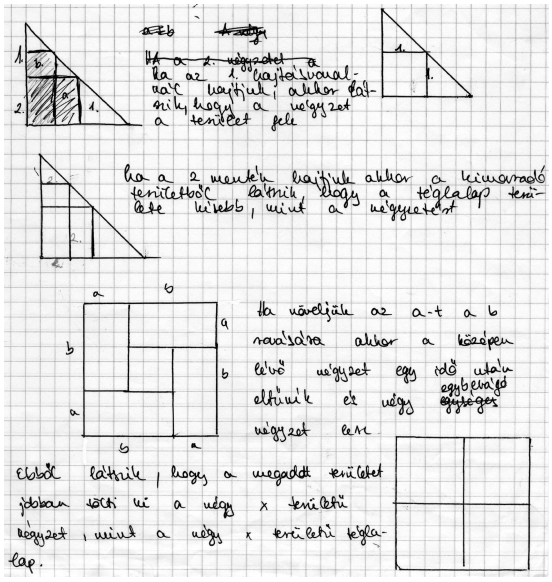

Figure 3. - Modelling

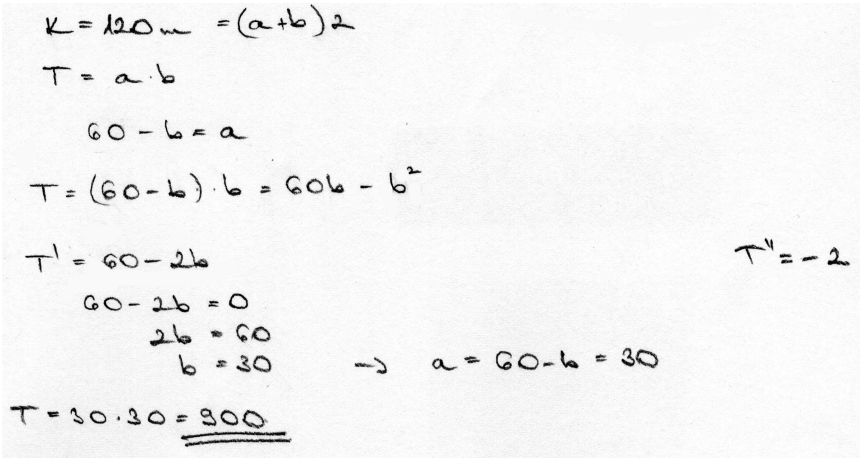

Figure 4. - Method of derivation

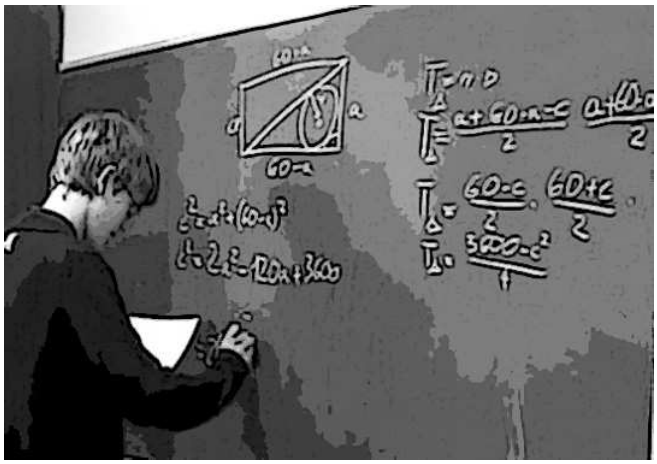

Figure 5. - Finding a non standard geometrical method

Realization of the experiment

In this part we will present and discuss three lessons at grades 9, 10 and 11. We investigated the solving process in three schools. We will choose and present some episodes from our observations.

An important goal of the lessons was to find more solutions of the posed extremal problem.

We saw that the starting of the lessons, the teachers' instructions and their behaviour were different.

We saw that the method of group work was determined mainly the teachers' instructions and slightly the individuality of the students.

High school Fazekas (grade 9)

The teacher gave the problem on a paper sheet to each group. The teacher observed the work of the groups and talked a lot with them and helped them in their work. The members of the groups discussed together the problem, tried to think and to solve it.

High school Református (grade 10)

The teacher made known the problem telling the text of our task at first and then she gave a paper sheet with the problem on it to each group. She suggested the students of the class working alone for several minutes.

High school Kossuth (grade 11)

The teacher did not read the text but after organizing the work he only gave out the papers. Students had read our problem alone, they started individually. After finishing the task they spoke about their solutions gladly both with the other students and the teacher.

The teacher gave a generalization of the problem. He supposed that certain circumstances the farmer has to build only three sides of a rectangular area. The students have to find the maximum area under these conditions. Then the teacher modified further the problem. At that time the farmer had to enclose only two sides of the area. The conjectures of the students were correct concerning the change of the area at first. But there was not an immediate conjecture for the second case. For this reason this last problem became a home-work of the students.

Conclusions

The completion of the experiment verified our initial conceptions.

- **Successfulness**: each group could solve the extremal problem in some kind of way.
- **Strategies**: we observed during the experiment that the strategies of the solving process depends not the age but the features of schools (teacher, students' ability and interesting, social conditions of students, etc).
- **Sex**: we could say that both girls and boys solved our problem, showed their solution to the others, apart from the fact that the teacher was a man or a woman.
- **Choose**: there will be groups consisting of only boys, only girls, one girl and three boys, one boy and three girls, etc.

Our observations confirmed the following statements of the didactical literature. The research clearly indicates that cooperation, compared with competitive and individualistic efforts, typically results in

(a) higher achievement and greater productivity

(b) more caring, supportive and committed relationship

(c) greater psychological health and social competence

(d) self-esteem.

The positive effects that cooperation has on so many important outcomes makes cooperative learning one of the most valuable tools educators have. We saw that the attitude of the teachers was different. Some of them was surprised from it, the other was accostumed to the Hungarian teaching style but he tried to apply this method.

We put a question not only to the teachers but to the students after lessons. The 90% of the students think that the cooperative learning is much better than the Hungarian traditional education which has a Prussian origin. In the 10% of the students' opinion these lessons were more interesting but less effective than the others. Mainly the good students of math are averse to the cooperative learning. One student wrote down that she was disturbed by the video camera.

We should like to develop the use of the cooperative learning in Hungarian schools further on. And because of this after a detailed analysis of this teaching experiment we will acquaint teachers with the bases of this method. It is our belief that it would be very useful for - with the exception of the best students - almost everyone. We could increase the effectiveness with initiating PISA-like problems in the experiment. These application-oriented tasks - through the increasing of the student's motivation - could also increase the common manipulations of groups. Beside students we have to pay attention to the teachers, too. We will analyse a lot of lessons with the help of the video technics and our teachers.

Literature

[1] Bartha, G. - Bogdán, Z. - Csúri, J. - Duró, L. - Gyapjas, F. - Kántor, T. - Pintér, L.: *Matematikai feladatgyűjtemény I.* Nemzeti Tankönyvkiadó Budapest, 2006.

[2] Busse, A. - Borromeo Ferri, R.: *Methodological reflections on a three-step-design combinig observation, stimulated recall and interview* ZDM Vol. 35 (6), 2003.

[3] Burns, M.: *The math solution: using groups of four* In N. Davidson (ed.): Cooperative Learning in Mathematics - A Handbook for teachers. Addison-Wesley Publishing Company, 1990.

[4] Clarke, D. - Wilson, L.: *Valuing What We See* The Mathematic Teacher Vol.87, No.7, 1994.

[5] Crabill, C. D.: *Small-group learning in secondary mathematics classroom* In N. Davidson (ed.): Cooperative Learning in Mathematics - A Handbook for teachers. Addison-Wesley Publishing Company, 1990.

[6] Davidson, N.: *Introduction and overview* In N. Davidson (ed.): Cooperative Learning in Mathematics - A Handbook for teachers. Addison-Wesley Publishing Company, 1990.

[7] Dennis, M. A. - Hamm. E. M.: *Portfolio Assessment and Social Studies: Collecting, Selecting, and reflecting on What is Significant* Social Education, 103-105., 1992.

[8] Garcia, N. - Herbst, P.G.: *Teaching mathematics with problems* University of Michigan, 2005.

[9] Horváth, A.: *Kooperatív technikák* OKI Budapest, 1994.

[10] Johnson, D. - Roger: *Using Cooperative Learning in Math* In N. Davidson (ed.): Cooperative Learning in Mathematics - A Handbook for teachers. Addison-Wesley Publishing Company, 1990.

[11] Johnson, D. - Roger: *Cooperative Learning* http://www.clcrc.com/pages/cl.html, 2001.

[12] Johnson, D. - Roger: *An overview of cooperative learning* http://www.co-operaton.org/pages/overviewpaper.html, 2001.

[13] Kagan, S.: *A kooperatív tanulás* Ökonet Kft., 2004.

[14] Koss, R. - Marks, R.: *The Teacher and Evaluation* The Mathematics Teacher, Vol. 87. No 8.

[15] Malara, N. A.: *Promoting Teachers' Changes: examples from an educational process in early algebra* proceedings of the International Conference, The Mathematics Education into the 21-th Century, University of Brno

[16] Pólya, G.: *Mathematical discovery* John Wiley, New York, 1962.

[17] Raymond, A.M. - Leinenbach, M.: *Collaborative action research on the learning and teaching of algebra: A story of one mathematics teacher's development* Educational Studies in Mathematics 41, 2000.

[18] Sapon - Shevin, M. - Ayres, B. J. - Duncan J.: *Cooperative learning and Inclusion* http://www.co-operaton.org/pages/overviewpaper.html

[19] Vygotsky, L.: *Thought and language* Cambridge, MA: MIT Press, 1962.

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