WHY IS THE SIMPLE MATHEMATICS SO EXCITING?

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ABSTRACT. *In this paper an attempt is being made to discuss what is the so called simple problems may appear exciting, simple and attractive.*

1. INTRODUCTION

The question is very simple, very serious and very exciting. We do not pretend to give an answer to such problem with which mankind is dealing from at least from the era of ancient Greece.
We will try to deal with the simplest question – why the mathematics at the very beginning can be so clear exciting and giving direction to the mind?
It is always nice if after some attempt you are able to achieve some even smallest success – which in mathematics is always possible and rather easy to document.

2. ABOUT ONE SPECIAL THREE-DIGITAL NUMBER

For example we could regard the following problem from the Lithuanian mathematical Olympiad. It is such a problem who is accessible for everyone who is eager to solve the problem, who is eager to make some progress in his mind and who can spare some time for solving of such a problem.
The other thing is that such a problems are of great value for the calculating oriented mind because it is from their field too – they are permanently being surrounded by numbers and the are dealing each day with numbers – very often with the huge ones.
Let us come back to the problem what we are beginning to discuss. We are given the three-digital number. It may be that such a number is divisible by 11 or not. For example 132 is divisible and 999, of course, not. Let us call that the number is hereditarily non-divisible by 11 if the number itself is non-divisible by 11 and any number which we can get from the initial number changing one of its digits to any other digit is non-divisible by 11 too.

At first this question seems not so easy. We need some working tools and that is in this case of course the criteria of divisibility by 11. Even not knowing anything about it is not so difficult to guess what it may look like. Regard some numbers who have 3 digits and are divisible by 11. The few least such numbers are 110, 121, 132, 143, 154, 165, 176, 187, 198, 209, 220. What the property has such a numbers in common? All of them except of one – namely 209 – have the property that the sum of first and third digits is equal to the second digit. And the sum of the first and third digit of number 209 is 11 and the second digit being 0 so their difference is 11. So the suspicion occurs that if the difference is 11, so that is good. Try further – 231, 242, 253, 264, 275, 286, 297 the same case with equal sums, then 308, again difference of the outside digits differs by 11 from the inside digit. Other examples could be 409, 508 and so on.
So for three digital numbers we may guess and in each book to find such criteria which appears to be right.

The number is divisible by 11 if the difference of sums of its "even" digits and of "odd" digits is divisible by 11.

Now we are eager to establish whether among three-digital numbers there are such who are hereditary non-divisible by 11.
For the very first example regard the least three-digital number 100. If we are allowed to change one digit of it in order to get the number which is divisible by 11 then that is easy – we may take 110. In a case of 101 we may take 121. And in general if the both outside digits have the sum not exceeding 9, then we can take the inside digit being equal to the sum of these outside digits and so we are done. For example in the case of numbers 102, 212, 382, 591, 423, 890, 257 we can choose the numbers 132, 242, 352, 561, 473, 880 and 297 who are divisible by 11.

The case with sum 10 we left for a while aside, the cases with outside sums from 11 till 18 are clear, for example 427, 597, 845, 986, 709, 899 and 959 may be changed correspondingly to 407, 517, 825, 946, 759, 869 and 979.

Finally the case with the sum of outside digits being equal 10 or something like cases 129, 199, 852, 288, 357, 783, 426, 684, 535, 545 and 555 remain. If outside numbers are 1 and 9, then for inside number being between 1 to 7 we can find such a number with the difference 11 – in our case 429 for the number 129. If inside digit being 8 or 9 just as in the case 199, than we take 198 with equal sum. If outside digits are 2 and 8 then for the inside digit being equal from 1 till 6 we can find a number with the difference 11 - in our case 858 for the number 852. If inside digit is 7, 8 or 9, then we take 286 with equal sums for the number 288. Again if outside

digits are 3 and 7 then for inside digit from 1 till 5 we find the number with the difference 11 - in our case we take 957 instead of origin 357. For the inside digits being 6, 7, 8 or 9 we take in our case 781 for the 783.
If inside digits are 4 and 6 then for inside digit from 1 till 4 we take e.g. 726 for 426 and for the cases 5 till 9 e.g. 682 for 684 with the sum of outer digit being equal to the inside digit.
If both outside digits are 5 then for the case of inside digits from 1 till 3 we make the difference 11 - e.g. 539 for the case 535 and for the inside digits 5 till 9 we arrange the equal sums e.g. 550 for 555.
It remain the case 545 in which we are not able to do anything by changing one digit and so this number is the only three-digital number who is hereditarily non-divisible by 11.

3. SOME REMARKS OF THE PHILOSOPHICAL AND HISTORICAL NATURE.

Some years ago the author proposed this problem for the reader of Lithuanian computer magazine asking to find actually this three digital hereditarily non-divisible by 11 number and asked whether there are others having more digits hereditarily non-divisible by 11 numbers. The readers easily found the number 545, which we actually discussed and disclosed another two numbers having more than three digits and also hereditarily non-divisible by 11. The most interesting thing about them was that both these multi-digital numbers can be written using only two different digits - that's why the following questions seem to be rather interesting:

1. Are there infinitely many hereditary non-divisible by 11 numbers?
2. Do they always have only 2 different digits?

Another matter why the problems of such a kind are of remarkable value is that such question are understandable and accessible for each person who wants to solve it and achieve a progress. What is even more valuable such one who wants to solve such a problem may simply take the calculator and begin to find such a numbers one by one or to proceed a simple procedure which would enclose and print all such a numbers.

4. **IN ONE CASE POSSIBLE AND IN OTHER SO SIMILAR - NOT!**

The are many problems having similar conditions and such that one of them has a real solution and another not. This is again very interesting while letting our to be able to disclose the truth became more perfect.

Here we will discuss one problem of such a kind who we've taken from the Belarus Olympiad.

Do there exist such four real numbers a, b, c and d such that all theirs possible mutual sums would provide to the set of numbers:

A) 1, 2, 3, 4, 5, 6?

B) 1, 2, 3, 4, 5, 7?

Both conditions are so similar - these sets of possible mutual sums differ by only one - in our case the last element.

5. SOME GENERAL REMARKS OF THE PEDAGOGICAL NATURE

The following problem ought to be raised in this place.
The author all his life long was and actually is lecturing at the Vilnius University. For about ten last years author is teaching in the normal high school as well. This is not always easy but it provides to somehow let us say stereoscopic view concerning the state of education in general and the ideas how the teaching could be provided in general.
The author is fully aware of the situation at school concerning the situation when the student is able to guess, say, the solution of the equation without solving it or even without being able to do at all.
Guessing is in its human nature as natural as possible and is essentially speaking a art of prognosis or what the normal human person does this so many time every day in order trying to understand what is waiting on him.
Guessing was always the attempt to master the situation in which we actually are or soon will be.

Now after these simple remarks let us turn back to the initial problem.
When trying to guess an answer in first case after some minutes of simple experiment we would easily found the solution for the case A: or the four numbers 0, 1, 2 and 4 giving the set of their mutual sums 1, 2, 3, 4, 5 and 6. We could of course get it solving the corresponding equations but finding in the "wild way" is very honorable. Still we do not know whether these are all possibilities. In this a case there exists another one and we wish the reader success in finding it.

Trying to find the answer in another way by guessing we have no success at all - but strictly speaking it may mean nothing except of our non-ability to enclose the real state of things.
In order to clear the matter of things we are going to regard this situation in general.

Firstly we order them by magnitude noticing that no two of them can't be equal because of otherwise we wouldn't have 6 elements in the set of all possible mutual sums - and this valid for the case A as well as for B. Further assume that $a < b < c < d$.
It is clear that the least sum 1 is provided by the two least numbers a and b and the greatest sum 7 by the two biggest numbers c and d. In other words $a + b + c + d = 8$. Summing it in another way we get that $(a + b) + (c + d) + (a + d) + (b + c) + (a + c) + (b + d) + (a + c)$ is the same as the sum $3 (a + b + c + d)$ or is equal to the sum $3 \times 8 = 24$ or otherwise the same sum is $(1 + 2 + 3 + 4 + 5 + 7) = 22$ which gives us a contradiction showing that the finding of such a numbers is impossible.

6. IS IT POSSIBLE OR NOT?

Sometimes the fantasy of person able to provide mathematical investigations can be aroused by the problems of the kind asking: is it possible to do this or this or that? The usual answer no awakes the fantasy and eagerness to know - why is it impossible?

Let us regard as an example the following example taken from the Moscow mathematical Olympiads:
Around the circle the natural numbers from 1 till 12 are being written. Is it possible to write them down in such a way that the difference of each two of them is 3, 4 or 5?
If we would go along trying to arrange these just as asked we wouldn't be able to achieve the requested difference. So some suspicion arouses that this is perhaps not possible. This brings with itself the Hamlet's question – to be or not be?

In such a place we always are in the state of being of Gulliwer, who being tired or simply exhausted when sleeping in Lilliputianland awoke and find him being bounded very carefully to the earth. After he aroused and tried to rise up the situation was quite similar as in the case when we are trying to solve such a problem about which just before we have had no essential information.

Just trying to arrange the things in the proper way we will notice that the numbers which are neighbors in the usual sense - e.g. 3 or 4, or 4 and 5 are not neighbors any more. Trying to find some key to the solution of the problem we are able to state that the numbers who are far from and these who are very near one to another are far enough.
So we land in the set of numbers 1, 2, 3. 9, 10, 11 and 12 and state that no two of them can be neighbors in the sense with required difference.

Because there are only 12 of them that if they could not be neighbors they must be placed in each second place when being written around the circle. But then the remaining number 4 (or symmetrically 9) ought to possess two numbers among the named 6. But it has only one neighbor of such a kind. This is a contradiction and in fact not the easiest one.

7. FINAL REMARKS

Some persons regard the in our days the interest for the real mathematical knowledge is decreasing.
According to our understanding the situation is not simple but otherwise very promising because of the fact that the computers are distributing so large amount of numbers everywhere around.

On the other side the algorithmic knowledge that is how to do or arrange things is more and more requested.

This looks very promising if only we will be able to use and develop such a state of being.

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