TEACHING MATHEMATICS III

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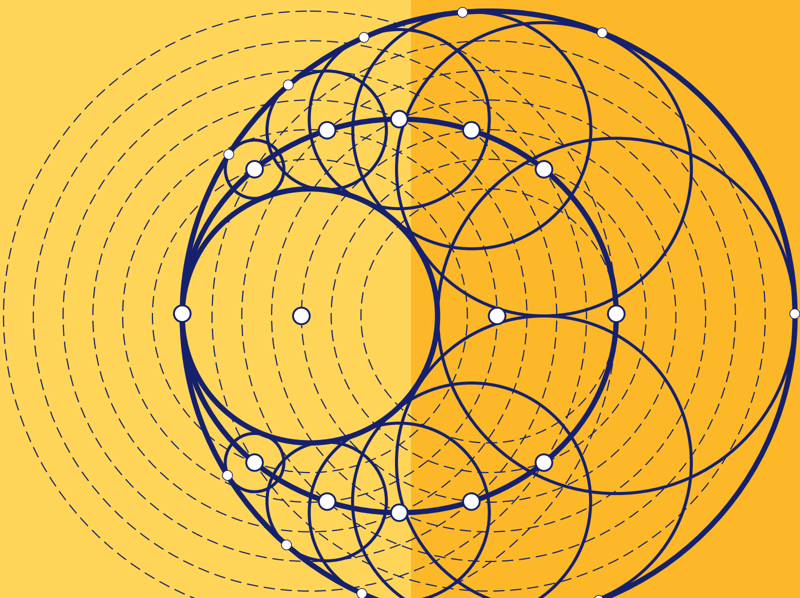

Innovation, new trends, research

TEACHING MATHEMATICS III: INNOVATION, NEW TRENDS, RESEARCH

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PREFACE

This volume of 16 texts presents the authors, who took part in the Mathematics and Informatics Didactics Conference, which was held at the Juraj Páleš Institute of the Faculty of Education of the Catholic University in Ružomberok, Levoča, January 20 – 22, 2012. The main goal of the conference was international exchange between researchers in mathematics and informatics education. The big support for the conference was the participation of the PhD students from the PhD School in mathematics and informatics education of the University of Debrecen, Hungary, Faculty of Informatics of the University of Debrecen, Academic Commission of the Hungarian Academy of Science in Debrecen, Kölcsey Ferenc Reformed Teachers' College and other researchers and teachers from universities in Visegrad countries. The participants presented their research results, and possibilities of future cooperative research.

The primary objective of *Teaching Mathematics III: Innovation, New Trends, Research* is to present results from some of these PhD students and other experienced researchers and teachers. It is nowadays important exchange information between these groups of researchers, because PhD students bring new methods (for example in ICT aided education) and experienced researchers bring experience and good practices for the teaching. We hope that potential readers (teachers and researchers in mathematics and informatics education) of this publication can find inspirations for their educational and research work.

Martin Billich

Basic trigonometric formulas in an inquiring approach

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Abstract. The main aim of this paper is to present an inquiry based professional development activity about the teaching of basic trigonometric relations and some conclusions about possible implementations in the framework of regular school lessons. The activity itself was designed to understand basic facts about the structure and the construction of standard curricula parts and in the same time to achieve a higher consciousness level in choosing teaching attitudes.

Keywords: basic trigonometric formulas, inquiry based learning.

Classification: G60; D50.

1 Introduction

In the recent educational trend the introduction of inquiry based learning into day to day teaching practice has become an important goal ([5], [3]). However the real world approach based on modelling and inquiry steps was traditionally an organic part of mathematics teaching (not as a pedagogical method, but as a strategy for understanding [2]) the politicians and decision-making system needs new proofs for the efficiency and reliability of this approach ([6]). But to obtain demonstrative arguments a more organic structure is needed in the construction of curricula. This structure has to deal not only with pieces of content as independent entities but it has to deal with the teaching method itself, the activities students have to perform in order to achieve a deep understanding of mathematics as a science, as a human activity, as a useful viewpoint in handling various situations, a viewpoint that ultimately models human behavior. On the other hand teachers needs new materials for the implementation of IBL and moreover they need transformative training in order to get confidence in using IBL.

As a part of the FP7 project PRIMAS[1] the Babeş-Bolyai University organized several training courses on the practical aspects of inquiry based teaching of mathematics. This paper is a report of one session which was focused on the teaching of trigonometric formulas. The main aim of this session was double: to get a deep insight into classroom processes, learning difficulties, teacher attitudes, student reactions which can appear during the teaching of trigonometric formulas and on the other hand to develop an organic structure, suitable for IBL activities at classroom and at professional development level. According to these targets the session was divided into two major parts. In the first step of part I. (about 1,5 working hours) the participating teachers worked in small groups and each small group had to construct a detailed teaching approach with problems and proofs. To avoid similar or identical approaches at the beginning there was a brief brainstorming to emphasize all possible ideas and then each group had to work on a specific idea he has chosen. As a second step (about 1,5 hours) there was a debate on the constructed proofs, on the different approaches from the point of view of teaching practice. In the second

[1]Promoting Inquiry in Mathematics and Science Across Europe, http://www.primas-project.eu

part the teachers had to construct a new curriculum structure for this content unit based on IBL activities and real world application. For this part a consultation with several professionals (topographer, astronomer, web designer, architect) was used and after these consultations the teachers had to prepare a common structure. This report is focused on the first part, where teachers had to develop and to reflect on possible proving strategies for the basic formulas.

2 Proofs and strategies

Problem 1 *Construct a strategy to calculate* $\sin(x+y), \cos(x+y)$ *and* $\text{tg}(x+y)$ *if you know* $\sin(x), \cos(x),\ \text{tg}(x)$ *and* $\sin(y), \cos(y), \text{tg}(y)$.

The participants agreed after several comments that only geometric proofs will be considered because only this can guarantee a strong connection to students' former knowledge (in Romania trigonometry is taught in the 9^{th} grade, for 15 years old students). Some other approaches can be found in [10], these can be used to connect trigonometry and vector geometry, or complex numbers, but first the basic formulas need to be derived by elementary arguments. This shows that for a succesfull calculation (or proof) we need to construct a figure where we have the angles x, y and $x+y$ embedded into right angled triangles. After a short brainstorming the configurations from Figure 1 were found.

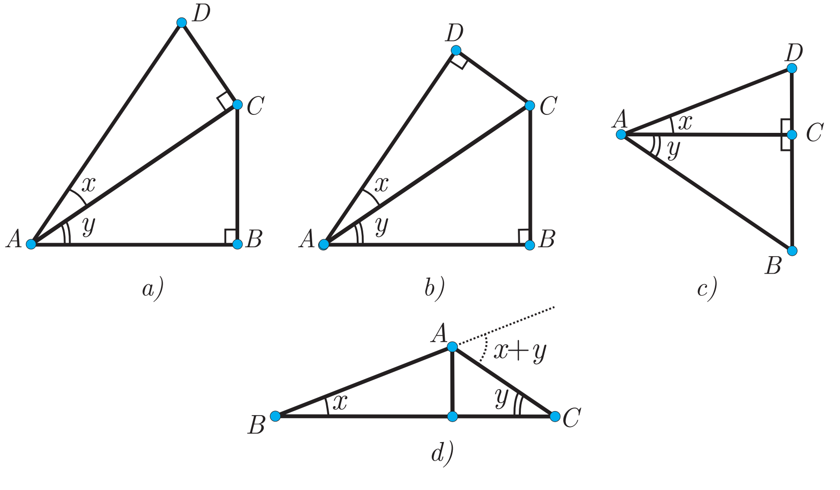

Figure 1: Possibilities to illustrate x, y and $x+y$, where x, y are in right angled triangles

Each group had to choose a configuration, to construct a proof and to analyze possible other proofs based on the chosen configuration.

Configuration a) We need to embed the angle $x+y$ into a right angled triangle. For this we have several possibilities (in fact there are infinitely many, but only a few are connected to the already constructed configuration). Hence we have to deal with the possibilities illustrated on Figure 2 (we omitted the case when a perpendicular to AD is drawn through C).

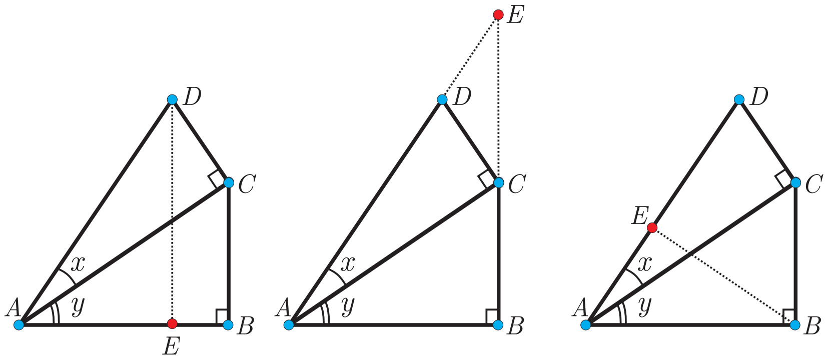

Figure 2: Possibilities to embed $x + y$ into a right angled triangle

Case 1. We use the first configuration in Figure 2. In order to be able to calculate the lengths of segments in the figure we need a reference length, which can be any one of the existing segments. For the simplicity we choose $AD = 1$. The strategy for calculation is very simple. We calculate the length of each segment using x, y and AD. We can fix $AD = 1$. Hence $DC = \sin x$, $AC = \cos x, BC = \cos x \sin y$ and $AB = \cos x \cos y$. To calculate DE we construct the orthogonal projection of C to DE. With this construction we have $m(\widehat{CDF}) = y$, so $DF = \sin x \cos y$. On the other hand

$$\sin(x + y) = \frac{DE}{1} = DE = DF + FE = DF + BC = \sin x \cos y + \cos x \sin y.$$

Using a similar argument we obtain

$$\cos(x + y) = \frac{AE}{1} = AE = AB - BE = AB - FC = \cos x \cos y - \sin x \sin y$$

and

$$\mathrm{tg}(x + y) = \frac{DE}{AE} = \frac{DF + FE}{AB - FC} = \frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y} = \frac{\mathrm{tg}x + \mathrm{tg}y}{1 - \mathrm{tg}x\mathrm{tg}y}.$$

Figure 3 illustrates the necessary constructions for the second and the third configuration in Figure 2. For completeness we reproduce the proofs for these figures too.

Remark 1 *This proof can be found in many textbooks and online resources. See for example* [2], [10].

Case 2. On the second configuration in Figure 3 we fix $AD = 1$ and we have $FC = \sin x \cos y$, $DF = \sin x \sin y$, $DE = \dfrac{\sin x \sin y}{\cos(x + y)}$ and $EF = \sin x \sin y \,\mathrm{tg}(x + y)$. Hence

$$\sin(x + y) = \frac{BE}{AE} = \frac{\cos x \sin y + \sin x \cos y + \sin x \sin y \mathrm{tg}(x + y)}{1 + \frac{\sin x \sin y}{\cos(x+y)}}.$$

From this equality we obtain (after rearrangement)

$$\sin(x+y) = \sin x \cos y + \sin y \cos x. \tag{1}$$

The formulas for $\cos(x+y)$ and $\operatorname{tg}(x+y)$ can be obtained in a similar manner.

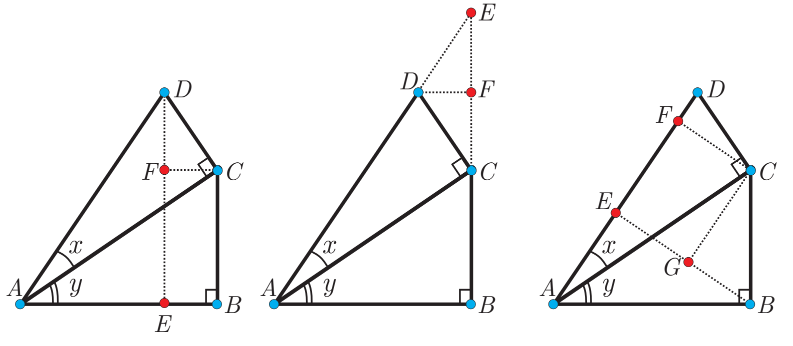

Figure 3: Auxiliary lines and calculation strategies I.

Case 3. We use the third configuration in Figure 3 and we consider as in the previous cases $AD = 1$. In this construction we have $m(\widehat{FCD}) = x$ and $m(\widehat{GBC}) = x+y$, thus

$$\sin(x+y) = \frac{GC}{BC} = \frac{EF}{BC} = \frac{AD - AE - FD}{BC}.$$

On the other hand $AE = AB\cos(x+y) = \cos x \cos y \cos(x+y)$, $FD = DC \sin x = \sin^2 x$ and $BC = \cos x \sin y$, so we obtain

$$\sin(x+y)\sin y + \cos(x+y)\cos y = \cos x \tag{2}$$

In order to obtain another relation between $\sin(x+y)$ and $\cos(x+y)$ we write $EB = AB\sin(x+y) = EG + GB = FC + BC\cos(x+y)$ so

$$\sin(x+y)\cos y - \sin y \cos(x+y) = \sin x. \tag{3}$$

Multiplying both sides of (2) with $\sin y$, both sides of (3) with $\cos y$ and adding the obtained relations we deduce 1 and from this we can obtain the corresponding formula for cosine and tangent.

Configuration b) Using configuration b) for the embedding of the angle $x+y$ into a right angled triangle we have the possibilities illustrated in Figure 4 (for each case we illustrated the auxiliary construction too).

According to these constructions we have to analyze the following three cases.

Case 1. We use the first construction on Figure 4 and we consider $AC = 1$. $m(\widehat{FDC}) = x+y$, so $FD = DC\cos(x+y) = \sin x \cos(x+y)$, $FE = BC = \sin y$ and hence

$$\sin(x+y) = \frac{DE}{AD} = \frac{\sin x \cos(x+y) + \sin y}{\cos x}.$$

From this relation we obtain

$$\sin(x+y)\cos x - \cos(x+y)\sin x = \sin y \tag{4}$$

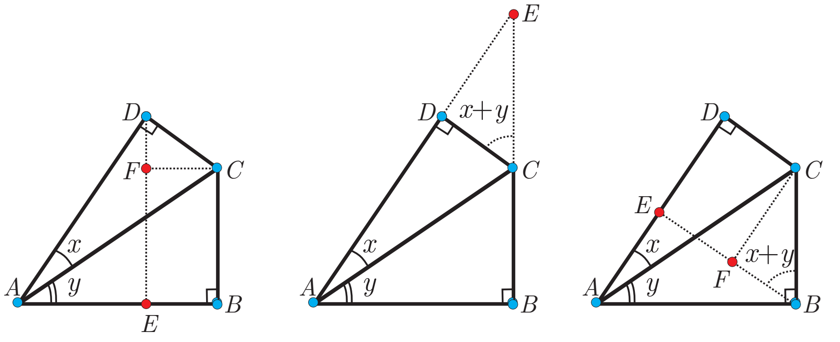

Figure 4: Auxiliary lines and calculation strategies II.

On the other hand $AB = AE + EB = AE + FC$, hence

$$\sin(x+y)\sin x + \cos(x+y)\cos x = \cos y \tag{5}$$

From relation (4) and (5) we obtain the well known formulas for $\sin(x+y)$, $\cos(x+y)$.

Remark 2 *It is possible to use a slightly different argument in this case. In the triangle ADE we have* $\sin(x+y) = \dfrac{DE}{AD} = \dfrac{BC+DF}{AD} = \dfrac{\sin x\cos(x+y)+\sin y}{\cos x}$. *From this relation we have*

$$\cos^2(x+y) = 1-\sin^2(x+y) = 1-\frac{\sin^2 x\cos^2(x+y)+\sin^2 y+2\sin x\sin y\cos(x+y)}{\cos^2 x,}$$

hence we obtain a quadratic equation for $\cos(x+y)$:

$$\cos^2(x+y) + 2\sin x\sin y\cos(x+y) + \sin^2 y - \cos^2 x = 0.$$

The discriminant of this equation is

$$\Delta = 4\sin^2 x\sin^2 y - 4\sin^2 x + 4\cos^2 y = 4\cos^2 x\cos^2 y,$$

thus

$$\cos(x+y) = -\sin x\sin y \pm \cos x\cos y.$$

In the last relation the last term could not have negative sign because $\cos(x+y)$ *must be positive, hence we obtain the formula for* $\cos(x+y)$ *and from this we can obtain the formula for* $\sin(x+y)$ *and* $\operatorname{tg}(x+y)$.

Case 2. Suppose $AC = 1$. $BC = \sin y$ and $EB = \cos y\operatorname{tg}(x+y)$ and from the triangle DEC we have $EC = \sin x\frac{1}{\cos(x+y)}$. From these relations we have

$$\sin y = BC = EB - EC = \operatorname{tg}(x+y)\cos y - \sin x\frac{1}{\cos(x+y)},$$

or

$$\sin x + \cos(x+y)\sin y = \cos y\sin(x+y).$$

Squaring both sides and using $\sin^2(x+y)+\cos^2(x+y)=1$ we obtain the same quadratic equations for $\cos(x+y)$ as in the previous remark, hence the rest of the proof is the same.

Case 3. As in the previous cases we suppose $AC=1$. From $AD=AE+ED=AE+CF$ and $DC=EF=EB-FB$ we obtain the relations (2) and (3), hence the rest of the proof is similar to the third case of configuration a).

Configuration c) Working with this configuration allows to use different tools. If we calculate the area of the triangle ABD in two different ways, we obtain a formula for $\sin(x+y)$. Suppose $AC=1$. On one hand we have

$$\sigma[ABD]=\frac{AC\cdot CD}{2}+\frac{AC\cdot BC}{2}=\frac{\mathrm{tg}x+\mathrm{tg}y}{2}.$$

On the other hand

$$\sigma[ABD]=\frac{AD\cdot AB\cdot\sin(x+y)}{2}.$$

From these relations we obtain

$$\sin(x+y)=\frac{DC+BC}{AB\cdot AD}=\cos x\cos y(\mathrm{tg}x+\mathrm{tg}y)=\cos y\sin x+\cos x\sin y.$$

Remark 3 *The use of the are can be avoided if we construct the perpendicular* BE *from* B *to* AD*. In this case* $BF=\mathrm{tg}y\cos x$ *and* $EF=\sin x$*, where* F *is the projection of* C *to* EB*.*

Configuration d) For the last configuration we can use several auxiliary constructions (if we want to embed the angle $x+y$ into a right angled triangle) or we can use an argument based on the area of the triangle (for this last argument see also [8]). We analyze two auxiliary constructions. These are illustrated in Figure 5.

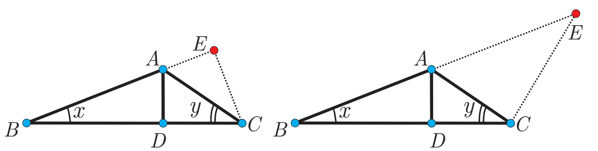

Figure 5: Auxiliary lines and calculation strategies III.

Case 1. Suppose $AD=1$. With this assumption we have $BD=\dfrac{1}{\mathrm{tg}x}$, $DC=\dfrac{1}{\mathrm{tg}y}$, $AC=\dfrac{1}{\sin y}$ and $EC=\left(\dfrac{1}{\mathrm{tg}x}+\dfrac{1}{\mathrm{tg}y}\right)\sin x$. From the triangle EAC we obtain

$$\sin(x+y)=\frac{EC}{AC}=\sin x\sin y\left(\frac{1}{\mathrm{tg}x}+\frac{1}{\mathrm{tg}y}\right)=\sin x\cos y+\sin y\cos x.$$

In a similar way we can deduce the formulas for $\mathrm{tg}(x+y)$, $\cos(x+y)$.

Remark 4 *This proof appears in* [13] *and* [14].

Case 2. In this case we denote by a and b the length of CE respectively AE. From the area of triangle BCE we obtain

$$a\frac{1}{\sin y}+\frac{1}{\mathrm{tg}x}+\frac{1}{\mathrm{tg}y}=\left(\frac{1}{\mathrm{tg}x}+\frac{1}{\mathrm{tg}y}\right)\left(\frac{1}{\sin x}+b\right)\sin x,$$

so

$$\frac{a}{b}=\sin x\sin y\left(\frac{1}{\mathrm{tg}x}+\frac{1}{\mathrm{tg}y}\right).$$

On the other hand from the triangle ACE we have $\sin(x+y)=\frac{a}{b}$, hence we obtain the desired relation.

Remark 5 *Using more auxiliary constructions and other arguments we can obtain also some other proofs. As an example we sketch a proof based on Ptolemy's first theorem (for more delightful trigonometric ideas see* [9]*). In the configuration c) the quadrilateral $ABCD$ is inscribed in the circle with diameter AC, hence due to Ptolemy's first relation we have*

$$AC\cdot BD=AD\cdot BC+AB\cdot CD.$$

If we suppose $AC=1$ and we use the relations from case 1/a) we obtain

$$BD=\sin x\cos y+\sin y\cos x.$$

On the other hand $BD=\sin(x+y)$ (due to the sine law or using an auxiliary construction), hence we obtain the required relation.

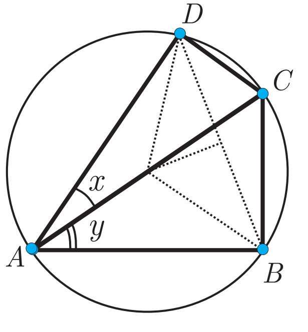

Figure 6: Auxiliary lines and calculation strategies IV.

Remark 6 *The formulas for $\sin(x-y)$, $\cos(x-y)$, $\mathrm{tg}(x-y)$ can be proved using similar arguments and figures. We propose to the reader as an exercise to analyze these geometrical proofs.*

Remark 7 *The main idea of the previous proofs, without a detailed analysis of the possible proving strategies, also appears in* [4].

3 Conclusions

1. Almost all solutions appeared at the training session (produced by the groups in the initial step, or at the presentation of the possible solutions), the author of this paper only unified and structured them. The participants were highly surprised about the diversity of possible approaches in contrast to the usual treatment of the problem in textbooks and classrooms.
2. The participants realized that basically all possible ideas were good and all ideas led to a correct solution (with more or less effort). By analyzing these proofs they realized that in many textbooks the presented solution is the shortest or the simplest as reasoning. This aspect is very important and shows that usually without analyzing other possible approaches, proofs, the main properties of a proof remain hidden. In most cases teachers present a possible proof, but they don't know that this proof is the optimal from some points of view. For this reason very often they can not answer the simplest question posed by students: "Why do we learn this proof?"
3. The diversity of proofs shows that if the necessity of these formulas appears as a problem in contextual problem situations, students have real chance to construct a geometric proof which is strictly based on their previous knowledge. Moreover by giving them the chance to construct such a proof and analyze some alternative approaches they can realize how mathematics works, how mathematicians work. They can understand that some tools can be more effective than others in constructing a proof and doing mathematics means that we seek for an optimal solution/proof (which is usually not unique because depends on the criteria we use).
4. Most of the proofs have one or two fundamental idea in the background, the rest are only technical details. The real understanding of mathematics starts with the understanding of these background ideas and the capability of making the connections between the simple and clear background ideas and formalism. All these proofs and ideas are useless in the existing Romanian curricula where first the trigonometric functions are introduced and the proofs are fitted to this approach (while this functional viewpoint is strange for most of the students). This shows that if we want to use IBL methods and activities, we need to completely rethink the structure of the content and in most cases we have to start with the problems (applications) that are at the end of the chapter in the present structure. The trigonometry represents an illustrative example in this sense. The geometrical applications are usually at the end of the chapter while fundamentally they motivate the existence of the whole chapter and they connect this chapter to the students' former knowledge.
5. The proofs presented here shows how inquiry based learning is working in an abstract framework and how simple ideas (the necessity of constructing the right angled triangles) combined with basic choices (the choice of configuration) lead to a correct proof. From this point of view the content itself in this activity is not as important as the processes the students/teachers are going through. This shift (from focusing on content to focusing on processes) is proper to inquiry based teaching however the importance of the content must be emphasized (by using an adequate context to introduce the problem and by giving other applications too).

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Programs for creating electronic study materials

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Abstract. The development of ICT brings new products constantly in technical and programs. Modern society is based primarily on the ability to obtain necessary information. An important task is now becoming an appropriate selection and processing of information, together with the manner of their use. Applications and procedures offering the creation of electronic materials large quantities on the market, however, not all are attractive and popular for users. Authors will be dealt with by several programs suitable for the production of electronic materials, and describe in more detail content management system WordPress in the contribution. Authors give a preview of the creation of an electronic material for computer networks at the and of the article.

Keywords: Electronic materials, software, chapter, WordPress.

Classification: R10, R70.

1 Introduction

A development in information and communication technologies is so rapid that it is way behind schedule seen in its entirety. With the development of new information and communication technology is the creation of study materials in electronic form.

The electronic materials created by different programs, must be processed to enable the student to understand maximum of the theme even without direct contact with the teacher. These materials require transparency, orientation and access to information.

There are study materials of different qualities, whose practical use can be disputed. It should be always good to consider the objectives in the creation of study materials, which we want to, achieve and create an appropriate study material, according to the instruction for the target group of users and the form of teaching. The creation of properly processed electronic study materials is not simple, and it is also time and financially consuming. Their creation is not only a technical issue, but also social. If we take the text originally in printed form, only in electronic form without other effects, it is such a little interesting and illustrative study material for the student. It must be just as substantially higher than in the printed textbooks in the electronic materials.

The use of multimedia electronic textbook presents a functional link of the text and the multimedia for the achievement of the effective promotion of the study. It is getting not only the content page of electronic materials in the foreground, but also their formal processing. There is currently available a lot of software resources for the creation of multimedia educational material. Creation at the high-quality products requires knowledge of the relevant software technologies.

2 Selected programmers for the creation of electronic materials

We will define some software products assigned for the preparation and production of interactive multimedia materials.

CamStudio is a free program, and the environment of the program is very simple. It consists of three separate, but functionally related programs. It's an application for recording video, which contains the necessary settings. Another component is SWF Producer. This is a program that converts video format AVI to SWF. The third part is the player AVI files. Its main task is to take action on entire the screen of the computer, or take action only of a part of the screen, that we showing a certain activity. *CamStudio* is a simple tool for creating multimedia materials, but is quite effective and fulfils the purpose.

Wink is freely available software for creating multimedia materials for e-learning. This application is available not only for the operating system Windows, but also for the operating system Linux. Unlike *Camstudio*, it is not necessary to examine through a number of text menu for each operation, but the icons of available functions are always located directly on the desktop program. *Wink* is most often used to create the animated tutorials, adding for example graphics, or spoken word, interactive elements (buttons for moving to a specific location in a presentation, or Web page). A standard output format of software *Wink* is an interactive Flash presentation. The software *Wink* provides the tools for the most demanding creators at a high level and the possibility to check the results of its work. The program also supports a file to export as PDF, HTML and PostScript.

Camtasia Studio resources include between commercial software. It is the complete solution for recording, editing and sharing videos in Web pages, CD-ROM and mp3. This software is available only for operating system Windows. In the processing of the video recording *in Camtasia Studio* editor, we can use special tags which may serve as identifiers for the time during the course or under them; we can divide or combine the whole video. The timeline contains a few feet, which are used for working with video and audio commentary, as well as for various special effects. The main advantage of this program is easy and intuitive environment for video editing, and exports the resulting presentations to create by the lot of supported formats. *Camtasia Studio* has the ability to shrink a record is from a larger resolution (e.g., 1024 x 768) screen to smaller (800 x 600), while maintaining readability and quality of output.

Another commercial product is the *Zoner Callisto*. *Zoner Callisto* offers tools for processing vector graphics. This program is among the appropriate instruments for home work with vector graphics and its creation.

Macromedia Authorware 7 is one of the most complex author systems for the development of interactive multimedia programs. It belongs to a group of paid products. It is the ideal tool for creating e-learning applications, interactive catalogs and publications, interactive training courses or all sorts of simulators. The scenario application creates on the timeline, where the icons are placed of the individual actions. Several icons represent the basic program structure, with the help of which it is possible to build very large applications. *Macromedia Authorware 7* makes it easy the integrated full-text search, to work with data on the Internet, the evaluation of responses, and the full support of e-learning standards AICC and SCORM.

ToolBook II Instructor belongs to the tools for creating high-quality multimedia materials. This is a tool that deals with the creation of electronic presentations and courses. The program provides courses in the form of presentation, in which it is possible to use an integrated scripting language. *ToolBook Instructor* has incorporated the link with the management system of Tutor2000, which is the product of firm Kontis Prague.

Commercial software *Macromedia Flash* is designed for creating interactive Web applications. Thanks to the use of vector graphics is the size of the resulting file not large. Bitmap graphics indicates for each pixel his position and color, and vice versa the vector graphics is registered as the mathematical equation. The use of sound in MP3 format and an Internet scripting language allows creating interactive multimedia presentations that are executable on all of the most popular browsers and operating systems. Flash animation may work with extern information stored, for example, in the classic text file, XML or PHP. Flash is mainly used for creating buildups and multimedia. Services of Google or YouTube make up data warehouses for video sequences, used consist of video players created in *Macromedia Flash*.

Pinnacle Studio is software with nonlinear content from firm Pinnacle Systems, which is a part of Avid Technology. It allows users to create DVD with menu and burn their video content, without the need for further additional software.

For example, ProShow Gold, Adobe Photoshop, CorelDRAW, Captivate, All Capture etc. belong to other commercial products intended for the production and preparation of multimedia of electronic educational materials.

3 The creation of an electronic textbook on subject computer networks with the use the WordPress

In the creation of electronic textbooks Selected chapters on the subject of the computer networks we used WordPress system, which allows, among other functions, to compile a simple and efficient electronic educational material directly in the Web browser interface. Electronic material is seen as a set of semantic-proper classification of chapters and subchapters.

WordPress is one of the most widely used content management systems for the creation of blogs, and small to medium-sized Web pages. The term blog, we understand the Web site, with frequent updates to the content. Its main advantages are easy to install, the transparency of the code, it is fast and free. Another advantage is the wide and active community of users, with resulting benefits, such as a large number of free templates and a comprehensive range of plug-ins available that extend the functionality of the platform. The basic package contains functions that ensure the immediate function of the system. On the Internet is a large number of plug-ins, which are usually available free of charge to WordPress. Easier plug-ins to just add some functionality, and more complex plug-ins added control panel. Knowledge of programming is not needed. In our work we used for WordPress system the following plug-ins:

- Easy Tube – makes it easy to add videos from YouTube,
- Tiny Style 0.0.1 – allows to edit the articles, parties, chapters using the built-in WYSIWYG editor TinyMCE Rich Visual Editor
- Tiny Table 0.0.2 – implements controls for the management of the tables in the text WYSIWYG editor TinyMCE Rich Visual Editor,
- wbQuiz 0.1.0 – used to this extension can be create simple tests,
- wp-table 1.52 – allows an extensive report tables

- WP Super Edit 1.1.2 – extends implemented WYSIWYG editor for many of the advanced text formatting features.

3.1 Add a new chapter

After opening Write it display empty form that contains a Write Post and Write Page. Write Post serves to add a new chapter, Write Page for the addition of a new test. If we want to add a new chapter in electronic materials, we write heading to form the element Title, such as "LAN Topology". To form element Post we add the text of the chapter, we can add videos, images and format text using a WYSIWYG editor. For the selection of the category to which the chapter belongs, we choose menu in the right hand menu called Categories. Specifically for the chapter "Topology LAN" we have selected a category of "Topology LAN". If we want to insert the video into the text of chapter from YouTube.com, just insert the address of the video. In an electronic textbook we used video Network topology bus (http://youtube.com/watch?v=sIkCwoEJyCE).

When we want to insert images to the text of chapter, we will use this form named Upload. We can load new image to that chapter, which currently fulfill.

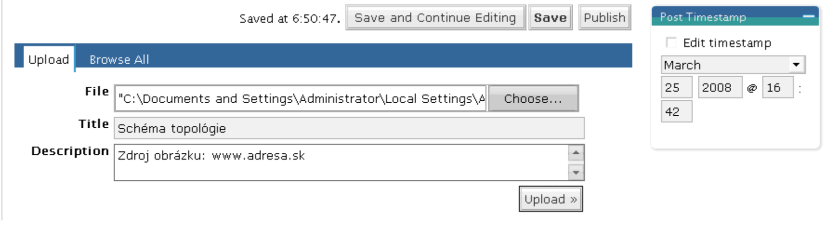

Figure 1: Upload new picture

It is possible select previously recorded images. Just choose the option Browse All in the form for the pictures and choose from them. The selection of the possibilities of inserting the image into the chapter is:

- Thumbnail – preview of the small appears to actual image
- Full Size – displays the image in full size
- Title – displays only the title to the image

Link to the image in the chapter may to be the following:

- File – you will see the image at full size in the browser window
- Page – displays the new page of the electronic textbook
- None – the picture does not click

After confirming the command Send to editor, the image loaded at the place where the cursor is currently in the text of chapters.

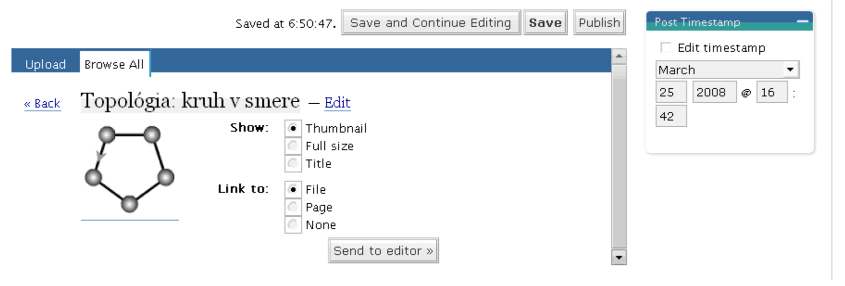

Figure 2: View an image in chapter

Chapter shall be made available to press Publish under article. The Save option saves and closes the chapter an editing form. Select Save and Continue Editing saves the chapter and continues its editing in the currently open form.

It is that specifies the ID attribute in table editing, the order the creation of the chapter and included unsuccessful attempts. Attribute When shows the time label, on which the chapter is published. The attribute Title shows the name of the chapter. Attribute Categories specifies into which categories the chapter falls. Attribute Comments shows the number of comments to chapter. The Author attributes show who is the author of the chapter. It is also possible to have multiple authors with different rights in the WordPress system. The last three items to View, Edit and Delete the whole record. View shows how the chapter look like in the browser, Edit opens an edit window of the chapter and Delete deletes a chapter.

3.2 Creation of a new test

If we want to add at the end of chapter for students test the knowledge of how the feedback, we must have activated the plugin in WordPress wbQuiz. If we have written a chapter from the LAN, we Manage the components of the Topology and your saved items Pages. To create a new test are two options, through the window and Create a new page link or by using the components of the Write Page, and its subdirectories.

We write the title and the name of the TextField to a form element to the Page Title. For clarity, it is best to choose a name that is similar to the name of the chapter to which the test belongs. The next step will be to select the menu from the right menu Page Template and then choose wbQuiz. This is a selection of templates for your page, to which we attach a file with the contents of the test. The next step is the choice of Custom Fields that we create the form Add below the contents. Individual form elements means:

- *Key- question_file:* key that is predefined by the plug-in wbQuiz. It serves for the party, which we create, use the plugin was able to currently.
- *Value-the wp-content/plugins/wbQuiz/test_topologia_lan*.php: marks the path to the content of the test, which is stored in this particular case in the file *test_topologia_lan.php*.
- *Action – Update/Delete*: after we enter the previous form fields Key and Value we can choose the option Update for changing an already existing test or modified test and select the option Delete to delete this field.

To save the page, which will contain the test, select the option Save and Continue Editing, Save or Publish. For our e-textbook we used php files with the names *test_transmission_media.php, test_wireless_networks.php, test_active_elements.php and test_end.php.*

It is in the beginning of each chapter clearly and comprehensibly defined the objective that contains the knowledge that they will be studying control after studying the whole chapter. It is indicated in color and located in a frame. After the goal, follow short introduction, which serves for putting up the themes of the subject.

In the introduction, it is appropriate to point out prior knowledge, which is necessary to control, so that we can continue to study. In each chapter it should be part of the motivation, which raises the interest in learning in students. It can be displayed in the form of a picture, animation, and video. We have chosen in LAN Topology chapter as motivation video from YouTube.com. Then new concepts, keywords, examples are following for a better understanding of the substance. All content is accompanied by images, or diagrams that students have also visually presented knowledge. At the end of the chapter it is a summary of the concepts to learn.

After studying each chapter is followed by the test. Students have the option to verify the acquired knowledge. In the tests, they used the open questions and closed questions with a choice of one or more correct answers. Each test contains an evaluation of the test. Evaluation provides students, but also tutors feedback on the gained knowledge. At the end of the electronic textbook is prepared a final test for the students, which includes questions from all the chapters.

4 Conclusion

A properly designed electronic textbook should have correctly defined goal, motivation, and include the requirements for previous knowledge. It is a definition of thesis, accompanied by illustrative images, diagrams, tables, which help students to better understand of the presented concepts to learn. At the end of the chapter processed is knowledge test, which verifies knowledge's from the corresponding concepts to learn, and provide students with feedback in the form results. Electronic materials have to compensate for lectures or interpretation and their expressing style, language, but also the graphic format must be well prepared. The form, in which it is submitted, plays an important role in teaching matters, because it replaces the missing school environment and the advantages of the presents study.

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Applications of informatics in teaching algebra

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Abstract. Since term of 2010/2011 I am a maths teacher in the Ferenc Kölcsey Grammar School in Debrecen. Due to being a night school the students' content is mixed because for attending to this school as adults there are several reasons. I knew the challenge of teaching there is different as in other high schools. My aim is to review my collected experience of teaching the topic 'Algebra and quadratic equation' and those handouts and instructional aids which were devised by me for maths lessons during the last and 2011/2012 semester. The students needed to have regular practice to acquire terms of algebra and get at good solutions of different tasks. For this reason I made two programs. Due to these programs my work became more effective and faster. Through these I can generate utility tasks and give them to students for homework. The hang of solution can be also generated by these programs so the checking can be simple. Through these exercises I can encourage my students for regular learning and practice. My experience shows that the students like this subject better and better and they solve the tasks more confidently.

Keywords: teaching algebra, informatics, application, mathematics.

Classification: D30; D40.

1 Introduction

There has been in the Ferenc Kölcsey Grammar School in Debrecen a night school education since 2003. The students have got lessons in two afternoons a week. At the present 350 students study in 10 classes. Our aim is to prepare them for successful final examinations. With 15 colleagues we try to do our best.

The number of mathematics lessons is three per week in the 9th form and two per week in the 10th form. The classes last 40 minutes. Because of this short time the students sometimes think we are on the run with curriculum. This and other reasons cause the so-called "small-sheet" method. In the next I detail this method.

1.1 Relation between the students and mathematics

In this grammar school the most students' aim is obtaining a graduate certificate. Only few students are motivated by furthur study. Those, who are in the lessons, really want to learn and acquire mathematics. Several students are in the classes after work thus they are tired even so enthusiastic. I want to maintain and support this inner motivation. Some students' desperation and will for learning are hindered by any former negative experience in maths. More students seem to be frightened of this subject because they think it is a mysterious and too complicated subject. They think it is impossible to acquire that. In the corridor and in the break I often heard the next: "Anyway I wish I could have two" or "I will never understand it". Therefore besides the acquiring of the curriculum It is very important to fight down their fears.

Although more students take an active part in the lessons their results are not so good as they were expected. From the tests it is clear that more students know the material even so they do not put up a good show. For getting good grades I try to make the lessons more informal. For instance the mistakes of the tests are discussed by the writer and the teacher. Through it I managed to achieve for the end of the semester that the students think of maths as it is a bearable subject and not so awful. Thus they do not have negative feelings because of maths lesson and there are not fears of failures.

Although the students are enthusiastically in lessons it does not mean the increase of knowledge or the acquirement of material but it is a very good base. For instance there is a 9th-form-student who had a bad attitude at the beginning of the semester and it had a big influence for his test results. However he was very active in the lessons. For the end of the semester he became one of the most interested and enthusiastic students. Now he studies in the 10th form. Before the winter break he asked if he could do more tasks from the book because he had an interest to practice more. So the positive attitude helps the learning process.

1.2 Topics in algebra

The algebra topics in the 9th and 10th form are the most significant because the students are expected to acquire the most important materials. In these classes they learn the role of letters and their formal thinking also improves. Although they have to memorize a lot of rules/axioms and recognize them during solving the certain tasks.

Topics in the 9th form:

- Using letters in mathematics
- Raising to a power
- Raising to an integral exponent
- Normal form of numbers
- Polynomes
- Nominal products
- Converting products with removing
- Converting products using nominals
- Calculations using algebraic fractions: multiplication and division
- Calculations using algebraic fractions: addition and substraction

Topics in the 10th form:

- Quadratic equations
- Formula of quadratic equation
- Radical form
- Relation between roots and coefficients
- Higher-degree equations brought into quadratic form
- Quadratic functions
- Quadratic inequalities

The topics above have got considerable importance to solve furthur problems and the topics of upper classes certainly rest on these materials. Learning how to solve those exercises which connect to the topics above is inevitable and it can be improved by practice.

The significance of algebra was worded by György Pólya and it is the next: *"The algebra is a language which consists of signs and not words. If we are experts on it, we can translate the certain sentences of daily life into the language of algebra".*

2 Problem posing and objectives

The students of grammar school have also got difficulties with topics above. They have often complained about it like this "there is no enough time for it", "we are on the run with topics", "I will practise more at home if you, teacher gives more exercises" or "In the lesson I understand how to solve it but at home I cannot do it".

One of the explanations for fast pace is that they attended to school years ago and now they have lessons once a week. For facilitation of learning I have done a helping material which contains solved tasks of different topics in details. The solution is shown step by step and with different colours. Due to the feedbacks this help seems to be very effective. In the furthur test there were some exercises from that helping material. Fortunately only a few tests seemed to be solved in the same way as it was in the helping material. Those students seemed to memorize the handout (20 pages) like a verse. A few students thought if they learnt the helping tasks word by word, they would know every task.

For showing the variety and opportunities of mathematics the topics of algebra was not finished although we had to continue the curriculum.

Checking of extra homework I want to maintain and encourage the students' approach to mathematics. It means more work and it raises more questions.

To improve the students' knowledge the lesson seems to be not enough because of the resons above.

Thus the next questions raise related to homework:

- *For one time how many exercises must I give them for homework?* I consider that how much homework is not too much and not too little but enough for practice; how much time the students need to solve the tasks from week to week; how to maintain their interests.
- *Do I give the same tasks for everone? Do they not copy the solutions? Do I give different tasks for everyone?* And this idea also can reflect the variety of mathematics.
- *How do I give assessments for their homework?* How to consider the wrong solutions? How to reward those who do the tasks well and who always work hard? Do I give "small five", red point or bonus, and etc. How will they respond these but they are adults?
- *Are there any appropriate maths compalation for them?*
- *How do I organize* the combination in a relatively short time and assessment of the tasks to be more effective?

Responding these questions led up to the "small-sheet" method which was named by students. Persistent work results in success, by which the students can be more encouraged and motivated in learning. Thus mathematics can be liked by students. Keeping in mind the main goal is to achieve a better learning process.

Solve the equations into the set of real numbers!

1.	$35x^2 - 128x + 84 = 0$	**11.**	$62x^2 - 157x + 38 = 2x^2 + 5x + 8$
2.	$10x^2 - 27x + 14 = 0$	**12.**	$109x^2 - 210x + 63 = -11x^2 - 5x + 8$
3.	$42x^2 - 199x + 14 = 0$	**13.**	$55x^2 - 101x + 90 = -x^2 + 3x - 6$
4.	$65x^2 - 157x + 72 = 0$	**14.**	$28x^2 - 532x + 1889 = x^2 - x - 1$
5.	$52x^2 - 211x + 60 = 0$	**15.**	$30x^2 - 51x + 17 = -6x^2 + 7x - 3$
6.	$7x^2 - 23x + 18 = 0$	**16.**	$83x^2 - 127x + 21 = 3x^2 + 9x - 3$
7.	$2x^2 - 25x + 12 = 0$	**17.**	$112x^2 - 426x + 134 = 4x^2 - 3x - 1$
8.	$11x^2 - 13x + 2 = 0$	**18.**	$105x^2 - 1633x + 1018 = -7x^2 + 7x + 10$
9.	$6x^2 - 11x + 5 = 0$	**19.**	$25x^2 - 43x + 33 = -3x^2 + 5x - 3$
10.	$9x^2 - 16x + 13 = 0$	**20.**	$84x^2 - 242x + 53 = 4x^2 - 6x + 9$

Figure 1: A worksheet

3 The "small-sheet" method

The students get 20 exercises, which are different per person. Because of economy in place two ranges of exercises are in one worksheet (A/4). In Figure 1 there is shown a worksheet of the 10th form.

As an assessment the students chose and insisted on the red points. They get these ranges of tasks every week and they have to solve them for the next lesson. If all exercises are correct they will get the red point. After getting five red points I give them a good mark. If there is only one mistake, they will have to do similar tasks. Only twice they have opportunity to solve them. If they do not manage to do them correctly, they will not get a red point. They must try to do it correctly. Those who do not deal with the tasks get a bad mark.

I am motivated to prepare and give different exercises per person so that the students cannot copy from each other and they can see also the variety of math exercises. Nevertheless there is a benefit if I have more and more exercises because later I will have a wide variety of worksheets, which I can use again and again.

Of course I do not forbid them to help each other or ask my help. But the main goal is to improve their learning therefore they must know what exactly the task is and so they can solve it alone and confidently.

4 Applications

There is only one unanswered technical question: *Where can I have the exercises from?* Those tasks which are in different maths compalations are not enough for that huge work. Preparing different exercises per student is a long and exhausting work. Therefore I had to think a lot about how to solve this problem. I always tell students to think before they want to achieve a simple way of solution. It is known that "mathematicians are lazy but programmers are lazier". Following this "theory" I was looking for an effective solving opportunity. I used my programming skills and I created an Applications which can solve the problem above. In the next I detail it.

This Applications was created in Java with NetBeans IDE 7.0. Modelling ranges of tasks was made with Microsoft Office Word 2007. For the automation I had to write some macro.

In the school-year 2010/2011 I used this program in tasks of 9th form and in the school-year 2011/2012 the program was used in tasks of 10th form. I prepared these two programs for own using therefore they have not got graphical surface. Thus we can work with them from standard input.

With parameters and changing the source-code the number of wanted ranges of tasks and the number of tasks can be modulated. By these data the program makes a simple word document which is formed into the wanted shape with Word macro.

4.1 9th form

In case of 9th form topics the tasks were created formally. I did not think it was important to give semantic interpretation thus the applications cannot help in checking. The number of polynomial expressions is variable.

During run the next exercises can be chosen:

- Monomial multiplied by polynomial, e.g.: $-2x^3y^3(2x^3y + 4y^4 - 3xy)$
- Polynominal multiplied by polynomial, e.g.: $(-5x^3y - 4x^4y^2 - 2x^2y + 3xy^4)(-2x^3y + 3x^4y^3 - 2xy) =$
- Removing, eg.: $20y^6 - 8x^4y^5 + 16x^2y^7 - 8xy^4 =$
- Binominal to the power two, e.g.: $(-3x^2 - 2y^3)^2 =$, and $4x^8 - 12x^4y^2 + 9y^4 =$
- Multiplying the sum and the difference of two terms, e.g.: $(8x^6y^7 + x^9y^6)(8x^6y^7 - x^9y^6) =$, and $64x^16y^14 - 36x^8y^18 =$
- Exponentiation and multiplication, e.g.:$4x^3y(-4x + y)^2 =$, and $4x^3y - 4x^2y^2 + xy^3 =$
- Multiplying the sum and the difference of two terms and multiplication, e.g.: $3xy^2(5x^4 - 4y^2)(5x^4 + 4y^2) =$, and $36x^11y^4 - 16x^3y^10 =$
- Mixed type of exercises: randomly chosen 2-2 from the former examples.
- Complex exercises, e.g: $3x^3(4x^3y^4 + 2xy^4 + 4y^2 - x^4) - (4x^44y^2 - 3y^3)(-3x^4y^4 + y) + 4xy(-2y^3 + 5)^2 =$

4.2 10th form

In the 10th form the aim of applications was making and solving quadratic equations and related exercises. Therefore it was indispensable to generate not only a syntactic visualized part but a solving part also. Thus the source-code became more complex and it could use the benefits of object-orientation by Java. The options are shown in Figure 2.

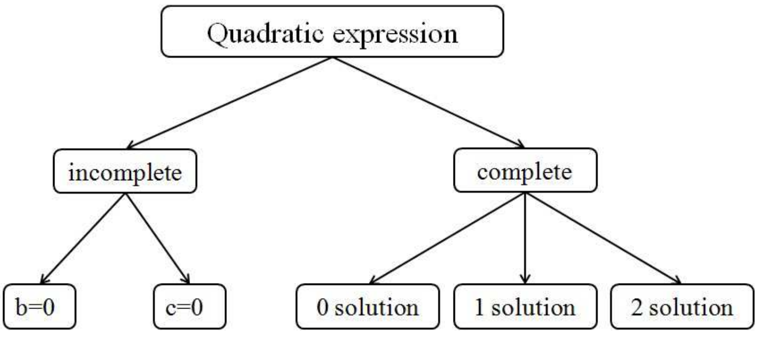


Figure 2: Quadratic Equation

By parameters it can be stated that the wanted equation is incomplete or complete. It can be given that the coefficient of the linear term or the rate of the constant is 0. In case of complete form the number of solutions can be also eligible. Also I paid attention to get a "nice" solution. It means that underneath the radical sign there is always a radicant. It seemed that the problem could not be solved by randomly generating of the three coefficients. Actually it is not the equation but the definition of its roots which is generated randomly. Then the roots are considered as fractions seperating the rates of numerator and denominator into the simpliest forms. Having the coefficient of the roots and the quadratic term the other coefficients can be calculated easily with Viète-formula. And it gives an equation: $ax^2 + bx + c = 0, a \neq 0$, where $a, b, c \in \mathbb{Z}$.

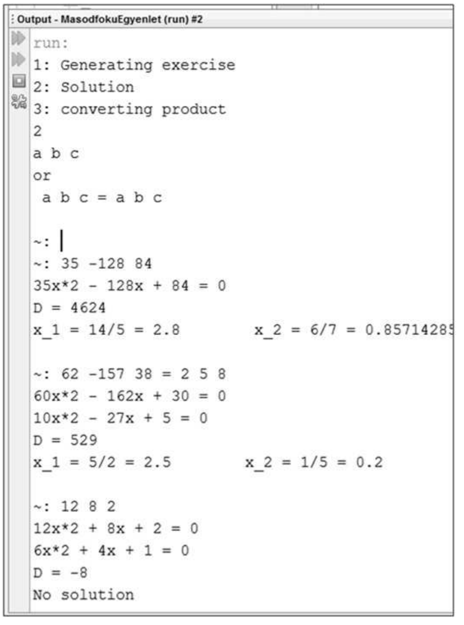
Of course before the students get the quadratic equations they have to be structured and then we can use the formulaof solution. And again I generate an unstructured version randomly with helping of two functions. Sometimes it can happen that the students have to work with huge numbers but the solution is acceptable.

In the case of converting into products and simplification of algebra fraction it is very important that the equation must be a root. In the case of fraction we have to consider that the two expressions must have common root.

The next expension is the case of quadratic inequalities. Here the challenge is rather how to prepare the solutions.

At the present the next types of exercises can be generated:

- Incomplete quadratic equation, e.g.: $x^2 - 8 = 0$, and $x^2 - 2x = 0$
- Solvable equation by formula, structured form, e.g.: $10x^2 - 27x + 14 = 0$
- Solvable equation by formula, instructured form, e.g.: $62x^2 - 157x + 38 = 2x^2 + 5x + 8$
- Converting products, e.g.: $10x^2 - 19x + 7 =$
- Simplify a fraction, e.g.: $\dfrac{9x^2 - 29x + 6}{9x^2 - 4x + 4} =$
- Applications of Viète-formula, e.g.: Without solving the equation answer how many roots the next equation have got and how much the sum and product of roots are! $x^2 + 4x + 11 = 0$



```
Output - MasodfokuEgyenlet (run) #2
run:
1: Generating exercise
2: Solution
3: converting product
2
a b c
or
 a b c = a b c

~:
~: 35 -128 84
35x*2 - 128x + 84 = 0
D = 4624
x_1 = 14/5 = 2.8        x_2 = 6/7 = 0.85714285

~: 62 -157 38 = 2 5 8
60x*2 - 162x + 30 = 0
10x*2 - 27x + 5 = 0
D = 529
x_1 = 5/2 = 2.5        x_2 = 1/5 = 0.2

~: 12 8 2
12x*2 + 8x + 2 = 0
6x*2 + 4x + 1 = 0
D = -8
No solution
```

Figure 3: Solution of Quadratic Equation

For faster checking and assessment the program displays the main stages of the solving process through giving the coefficients whether the form is structured or not. Thus we can have the partial results too. With it checking a worksheet - consists of 20 exercises - lasts for only 10 minutes.

After we start to use it we have to chose what we want to generate: either example or result. The solving process of exercises lasts until the user stops it. So there is opportunity to use the program during checking. It is shown in Figure 3.

5 Results

Firstly I tried the “small-sheet” method named by students and the way of practice above int he 9th form in the school-year 2010/2011. Then I have continued to use it in this class this year.

According to my experience the students solve the tasks with pleasure and enthusiastically. Every beginning of lessons they are excited whether they get or not red point, what the next range of tasks will be and their exercises are more difficult or their classmates’. The students chose one of the tasks which are on the table. They enjoy doing that even if their classmate is absent they chose for them too. I think its success is that they feel it more personal than the coursebook. They know those tasks are just for them. The students appreciate the work which is behind the exercises. Their enthusiasm is shown by that there were weeks when it was not necessary to have new range of tasks because of the material and the students were disappointed.

Until now there has not been any problem about that they do not do their own tasks. So I have not had to take sanctions. Sometimes it happened that they had not brought their tasks in time but they had good reasons to explain it. But as soon as it was possible they tried to make up leeway.

My opinion is that there is worth correcting their homework because the students' attitude is becoming better and better and they think of learning maths as a possible way and they attend to the lessons with pleasure.

Of course the learning of material is also a significant goal. We managed to achieve it. The students could solve the tasks easily and confidently for the end of the term. These results made me glad and I was expecting eagerly the next term (September, 2011). In that class there became more students. But the former students' results were very good and satisfying. During the review they could remember the solutions of tasks and there were no problems with converting into products. Only those who had not attended regularly for the lessons or who had just joined the class could have good results. After classes I tried to explain the material to them again and again.

I hope the same results will be solving quadratic equations and quadratic inequalities. It would be good when they have to solve a quadratic equation in their furthur study and they will be able to do it easily.

6 Conclusion

According to my opinion if there are exercises which are needed to practice regularly, the "small-sheet" method can be a long-term successful solution. Hereby the students have successful experience and they recognize how to achieve good results if they work persistently. Due to the methods above mathematics became liked by students. Later we can see how long-term their effectiveness are when the students have to use their acquired knowledge.

Due to informatics the teachers' huge work can be easing up. The range of tasks can be prepared and solved much more easily.

The development of disadvantaged pupils with GeoGebra

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Abstract. According to the tendency of the past 15 years, the number of students in primary schools is decreasing nationwide. Parallel to this, the number of disadvantaged students is increasing, exceeding one third of all students. This greatly affects students' value and behaviour brought into school. It is extremely difficult in case of these students to find a way that would lead to the development of self-motivation not only in the field of Mathematics but also that of general approach to studying. Within this, GeoGebra as a dynamic geometric system has proved to be very effective when developing disadvantaged students. I would introduce my observations in reflection of the past three years (2008–2011): the viewpoint for me was not only describing cognitive qualities quantitatively but I also emphasized the affective and psychomotoric factors.

Keywords: disadvantaged pupils, GeoGebra, spatial geometry.

Classification: G40, G80, C30, R20, U70.

1 Introduction

In the last 10-15 years, huge social changes took place in Hungary. The number of disadvantaged (and occasionally within this figure the number of multiple disadvantaged) pupils increased greatly. These changes of high extent implied the necessity of altering education-control. It could be regarded as a basic task to compensate the disadvantages originating from the social circumstances of these pupils as intensely as it is possible and prepare them to deal with the norms, psychological attributes expected by the society. [1] [3] The LXXIXth Law's 121.§ from year 1993 on public education in its "Interpretative Provisions" determines disadvantaged, multiple disadvantaged status as the following:

"Disadvantaged" child, pupil: a person whose right to receive regular child protection subsidy was determined by the notary, on the basis of family circumstances, social status; within this group the definition of multiple disadvantage is applied to: the child, the pupil whose parent maintaining legal supervision – according to his or her voluntary statement in the regulated procedure on child protection and guardianship administration – in the case of a kindergarten child, the child at the age of three, in the case of a pupil at the time of the start of compulsory education has maximally finished the eighth grade of school successfully regarding studies; multiple disadvantaged status is also valid for the child, pupil who has been taken into durable education.

According to some opinions the concept of multiple disadvantaged status was expressly created by the public educational code, which would designate the cause related to education (and only to education), which would lead to the child's social dependence. Namely that the parents' low educational qualifications are the indirect cause, why the family is socially in need for support: due to their low level of education there is no need to employ them on the employment market, or their income is so small that they reproduce unemployment through generations.

Rózsa Mendi [4] during one of her research, which brought very interesting results, has stated that the discrimination originating from the disadvantaged status (from society, classroom community...) is a serious psychological problem, and given of its nature it can be taken as an acquired cultural sense of shame. The consequences, symptoms given from this shame and internalized oppression continually intensify which would lead to the present conditions and changes in progress.

The critical problem, as we can see it above, has been already mentioned – in Hungary pupil numbers are decreasing but in parallel the number of disadvantaged pupils is increasing, so we can talk about proportionately significant changes (Table 1):

Primary School - statistical data			
School year	Total number of pupils (persons)	Number of disadvantaged pupils (persons)	Number of multiple disadvantaged pupils (within the disadvantaged) (persons)
2006/2007	831 262	217 328	61 494
2007/2008	811 405	228 349	85 798
2008/2009	790 722	241 739	100 119
2009/2010	775 741	257 335	106 539

Table 1: The development of primary school pupil numbers

This tendency can be extended to a number of other countries as well. This heavy social legacy was left to us mostly by the last 15 years. Since this process has already played out in some countries (for example in the US in the 70's), as a method of idea acquiring, the measures, methods that were introduced there in order to stop/reverse the process could offer a proper basis for us too, but other attempts, efforts have seen the light as well.

Some of these without the attempt of a full description:

- competence based education (EU Lisbon decision, 2000) competitiveness in the labor market
- new education methods (project, cooperative, student abstract...)
- differentiated occupational organizations
- Integrative Pedagogical System (IPS)
- Skill-development preparations
- Arany János programs
- digital education organization (education-organization supported by IKT tools, e-curriculum, the usage of teaching aids).
- ...

The basic educational task of mathematics would be to form psychological attributes in the pupils that are essentially necessary to fit in the society, and to offer an applicable knowledge which is a requisite for the people nowadays, supported by the model of lifelong learning.

A precondition for the above mentioned requirements is the formation of a proper motivational basis in the pupils. To create this in the case of disadvantaged pupils could mean an even greater challenge. According to my experiences (due to social/financial circumstances) the inclusion of the computer into mathematics classes creates this kind of a motivational basis among the pupils. Based on the following points of view the GeoGebra, as a development tool, is perfect for achieving the objective:

- The pupil's point of view: easy operation, lucidity, in Hungarian language.
- The educational institution's point of view: the software included in the teaching-learning process should be accessible for everybody for free if possible (in the institution and at home as well).
- The teacher's point of view: a simple user surface (teachability), at the meantime utilizing as many opportunities as possible, lots of aiding materials, developmentability, wide range of newer and newer possibilities.
- The educational management, mathematical-didactic and social changes of the last 10-15 years.

The motivation does not only have to be aroused but reinforced as well with planned, purposeful development. According to the opportunities of applying as many tools and methods as possible → we cannot know that in the case of individual pupils by what effect does the design of individual schemes make progress. [2]

2 Results and proofs

The schedule of the research is estimated to the period between August 2008 and June 2011 which means that the examination took 3 school years. The location was a primary school in a village, where we had to count with on the average 80% of pupils of Roma minority proportion yearly and with 60% of pupils with disadvantaged status proportion from year to year. This presupposed the Integration Pedagogical System's one level higher on the part of the institution, because of which some pupils received Skill-evolving development.

During the observations we must pay attention to that disadvantaged status does not always by all means walk in parallel with the "less talented" marker. Among the disadvantaged pupils a person can emerge who shows talent in some territory, so besides the compensational development we must be attentive to offer talent management fitting to his or her personality.

GeoGebra as a DGS, on the basis of my experiences that I gained in the last 5 years, offers an excellent opportunity during the development work and the nurturing of talents. The research strategy was realised in a control group experiment in which I held lectures for an experimental group using Geogebra too, (besides the traditional tools). The control group which worked without the inclusion of computers and GeoGebra, was lead by a teacher-colleague. This way we excluded the risk that we would even unintentionally influence the results for the benefit of either groups.

The research method is cyclically operationalized which involves the observation of both the quantitative (the examination of cognitive factors) and the qualitative (affective and psychomotoric) factors. Even though the second part is not held a significant point of view by many, I paid an emphasized attention to those factors as well because I found that especially in the case of disadvantaged children these could be very important factors to explore the real motives regarding mathematics and studying in itself.

The examination model was the current 8/a and 8/b classes, approximately with 20-20 pupils, where pupils in the 8/a class in the last 3 observed school years have fallen short compared to the pupils of class 8/b on the basis of input measurements with regards to performance. Each pupil was able to use a computer – thanks to a number of the institution's successful tenders, two computer rooms containing 20-20 computers were established.

I find it important to remark in what station do these pupils are with regards to psychological perspectives (on the basis of the division according to Piaget):

- Internalized concreteness: the child is able to imagine activities with concrete objects, which are not in his or her environment.
- Reversibility: the child is able to reorganize objects, and to imagine a concrete operation in a reverse direction.
- Decentralization: the child is able to focus on several aspects in the same situation and recognize connections.

Increasing security can be regarded as a main progress step in the execution of logical conclusions. Unfortunately my experience is that in the case of many pupils, not only does this development level delay, but it would not even take place ever. It is very important to pay attention to this point of view during inquiries, developments.

Before we started to develop with GeoGebra there were some previous problems that we had needed to solve. First, there was a high-level of undevelopment regarding the pupils' skills related to IT tools (the lack of PCs, mobile phones at home), this should be compensated during lecture even outside classes in co-operation with IT teachers and colleagues.

Secondly, before the introduction of GeoGebra development, we had to modify the schedule of development, according to a foreseeable curriculum. I did this at the expense of theoretical lecture/lesson-sections, because I think that manual activities are also very important besides virtual demonstration, since GeoGebra cannot substitute and never can fully replace the "take it to your hand" type learning, conceptualization. Pay attention to use GeoGebra implemented by the inventory of previously applied tools.

We have mapped out three possible dimensions of the pupils' disadvantaged status in the educational institution which served as the location of the research with the help of form-masters (Diagram 1).

70% of disadvantaged pupils live amidst difficult financial circumstances, furthermore it can be stated that 52% of them besides the financial circumstances is disadvantaged both from emotional and communicational perspectives. Of course the pupils' financial circumstances have a critical influence on their performances in the educational institutions: how can one expect the preparation of the homework from for example a pupil who has no desk, pencil or in a more radical instance food at home.

Besides cognitive measurements, I performed the examination of pupils' affective factors on the basis of a modified Claus-type questionnaire, which revealed the following:

- 76% of the pupils do not like mathematics.

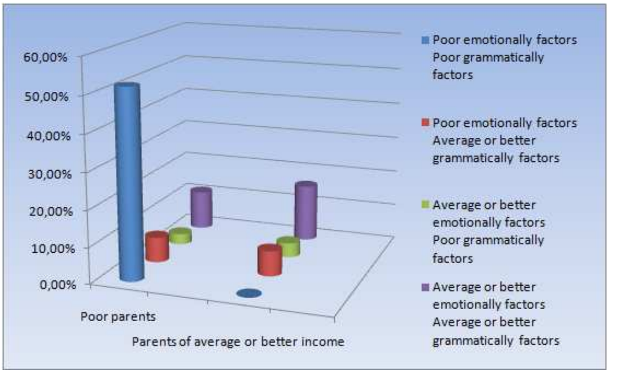

Diagram 1: The three possible dimensions of the pupils' disadvantaged status

- Most of the pupils do not like to solve geometric exercises.
- Most of the pupils do not like to solve more difficult exercises.
- 90% of the pupils are motivated by the use of computers on mathematics classes.

Last but not least, we must not forget about psychomotoric factors, in connection with which the main tasks are the following: [3]

- The clear, bright and simple record of task solutions.
- The skillful usage of callipers, ruler and other tools.
- Freehand drawing and sketching.
- Preparing models.
- The proper operation of computers.

With the traditional caliper-ruler drawings implemented with the GeoGebra, in my opinion we can meet the above mentioned expectations. In the following sections I will expound the tasks supported by GeoGebra topic by topic.

2.1 Pythagorean-Theorem

The Pythagorean-Theorem is one of the most determining theorem in primary school mathematics-education. Later this theorem has to be applied with several tasks which require more complex (spatial) solutions.

Two different worksheets will be demonstrated: one of them serves to confirm the theoretical background, implying to arouse the need for a correct demonstration (Figure 1).

The other worksheet tries to offer help through an everyday example regarding the application of the Pythagorean-Theorem (Figure 2).

By applying the dynamism and interactivity of GeoGebra, the pupils may change the various input parameters, by this confirming connections and indicating that mathematics is not a self-serving matter, but it surrounds us and it is present in nature.

2.2 Spatial Geometry

The development of spatial approach is very important for me, since in everyday life we live in space as well, and in many cases this space is reflected to planes (maps,

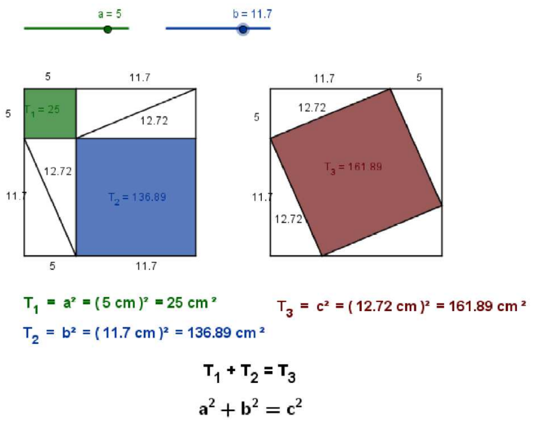

Figure 1: Implying the demonstration of Pythagorean-Theorem

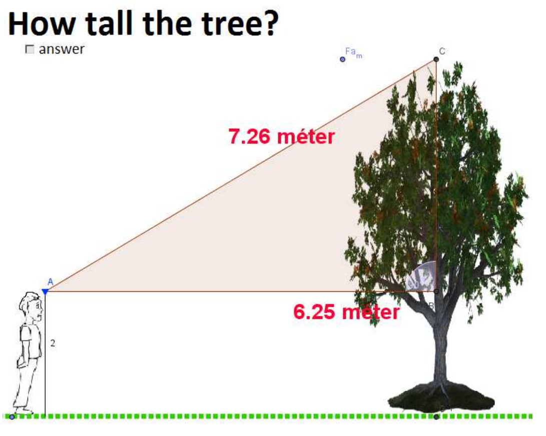

Figure 2: A possible application of the Pythagorean-Theorem

photographs, instruction manuals for tools...), this way it is vital that the pupil should be familiar with processes originating from space-time connections.

Regarding this topic, the following was presented with the help of GeoGebra:

- Grouping objects
- Different views of objects
- Network of objects
- Object surface, volume

A large group of GeoGebra worksheets, which serve to demonstrate different kinds of object views, can be found on the `http://dmentrard.free.fr/GEOGEBRA/Maths/accueilmath.htm` website. This is a fantastic collection which includes more than 300 GeoGebra worksheets that are related to space, and its application in the classroom proved to be very progressive (Figure 3).

It is worth to take a look at almost all of them with the pupils, they will enjoy it very much.

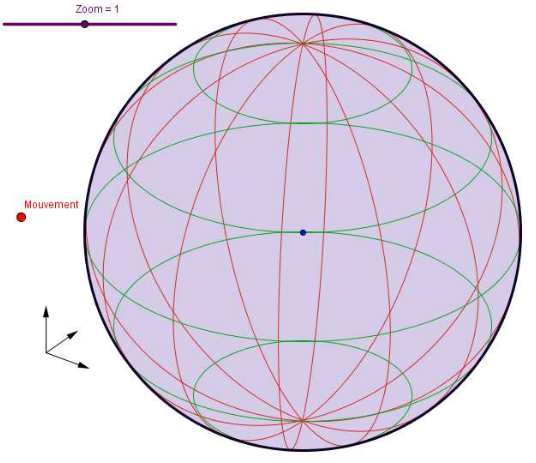

Figure 3: Displaying a sphere in GeoGebra 4.0

I have also applied self-made worksheets in this section (Figure 4).

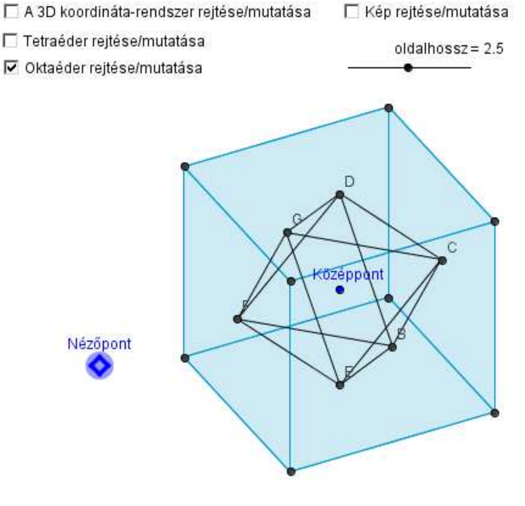

Figure 4: Rotatable cube and other spatial formations

These worksheets proved very useful regarding the discussion of the connections between different objects and their visualization opportunities: who would think at the sight of a square that it could even be a cube from a certain perspective.

The manual activities aided by GeoGebra proved to be more effective with regards to computations related to surface and volume too: more correct solutions were evaluated in the cases of such tasks in the experimental group than in the control group.

The survey closing the topic section indicated that from the perspective of the pupils' development, in the case of the experimental group, the worksheets, used to demonstrate the network of objects, have effectively supported the models that are handheld.

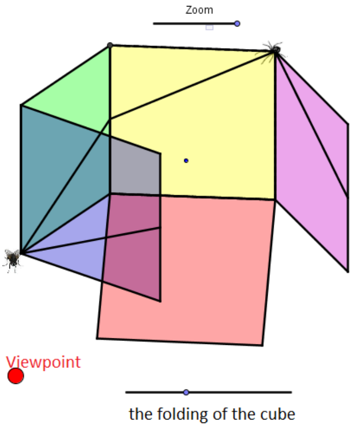

Figure 5: The case of the spider and the fly

I find it a very nice task to find the geometrical location of a point which is in motion in some way: the fly and the spider are in two opposite points of the cube. On which path could the spider get to the fly by moving on the surface of the cube? (Figure 5)

The measurements () prove that although the pupils' input performance from class 8/b fell a little short to the control group's performance, they managed to catch up and exceed the control group's performance related to certain spatial tasks.

2.3 Planar transformations

A number of well applicable worksheets are available for this topic on the official website of GeoGebra. I start this topic section among many others with a self-made playful worksheet which serves to confirm the concept of congruency.

Task: Match the formations that are congruent! Watch out, there's an odd one, too! (Figure 6)

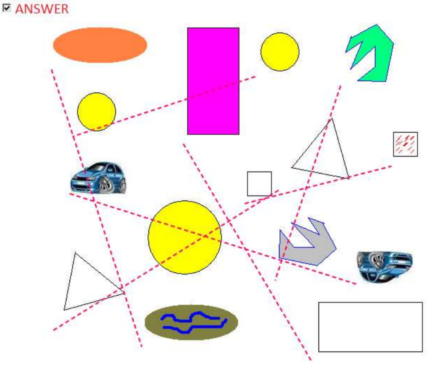

Figure 6: Match the formations that are congruent!

The self-verification can be realized with the help of a usual selection box. It is important to make the pupil aware of the definition of congruency that with regards to congruency's point of view, we don't care about color, texture or disposition, only about the form and the size.

After browsing the sheets found in GeoGebraWiki according to the topic section each pupil had to prepare his or her own GeoGebra worksheet which demonstrated an optionally selected planar transformation. Some of these pupil products can be seen below (Figure 7).

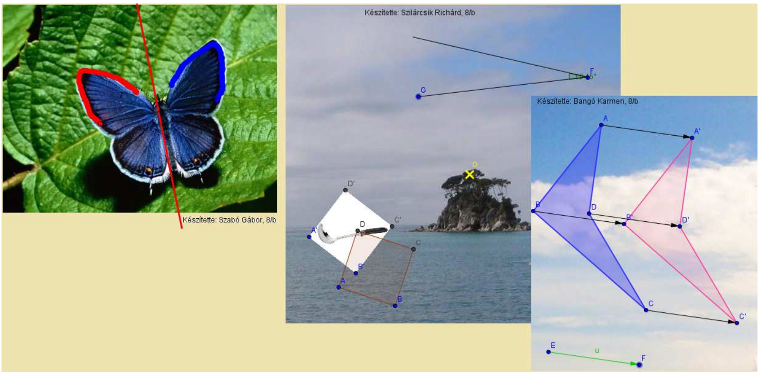

Figure 7: GeoGebra worksheets prepared by pupils

The independent completion of a task in GeoGebra helped the pupil to esteem his or her own and in the same time other pupils' work and it could offer a foundation to meet the expectations of nowadays IT society, namely the usage of IT tools, for which certain parts people have to discover on their own, without external help.

2.4 Functions

I divided the function topic basically to the following categories:

- The systematization of what the pupils learned about the functions' transformation
- Linear functions
- The absolute value function
- Quadratic functions
- Other functions (linear fractional functions, higher level functions, a mixture of previously known functions...)

GeoGebra offered great help for the pupils since we know about ourselves that if we work with a static figure (into a notebook or on the blackboard) then in the case of a single function the picture of the characteristic attributes, diagrams and the demonstration of different variable and value transformation is a quite lengthy process and it requires several pages of drawing.

Because of its DGS quality, within one worksheet, practically all possible dispositions of a function can be covered, or even more of them also with the objective of comparison (Figure 8).

In this topic section I alternately used GeoGebra and the traditional callipers-ruler drawing: in GeoGebra I demonstrated the images of the function for the pupils

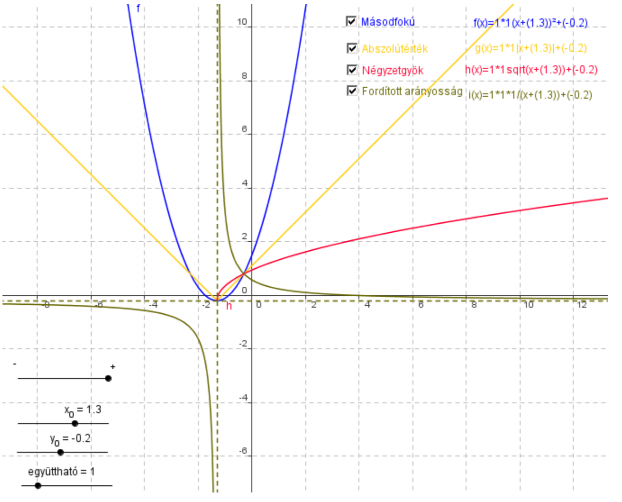

Figure 8: The relationship of various functions' transformed units

and in the notebook we performed several function rendition, after which we returned to GeoGebra again applying all of its advatages.

2.5 Preliminary examinations and results gained

In all cases input and the output measurements were identically concerned: the optimal acquisition of the curriculum defined by the skeleton curriculum was presupposed.

The following diagram (Diagram 2) characterizes the initial conditions, according to averages of input measurements, spread and relative spreads at the beginning of school years:

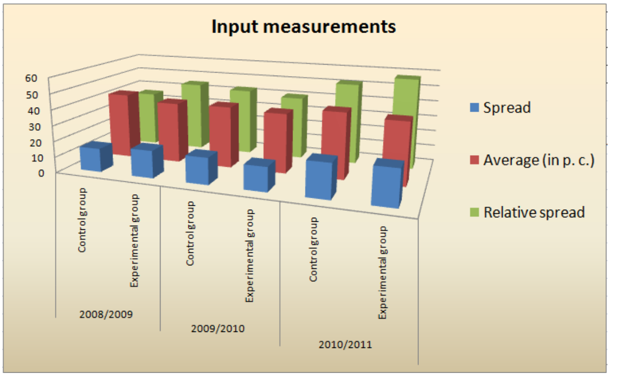

Diagram 2: Results of complex input measurements

On the basis of the diagram, we can observe that input measurement results show almost identical performance/ability groups, although some differences were seen every year: the experimental group performed less compared to the control group by 3-5% each year.

The following 3 diagrams (Diagram 3, Diagram 4, and Diagram 5) show the results of complex input and output measurement results broken down into school years compared to each other:

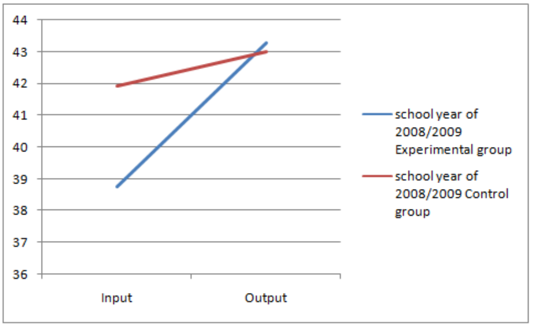

Diagram 3: Comparing measurements 2008/2009

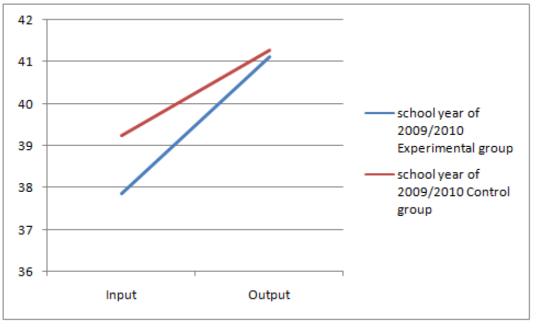

Diagram 4: Comparing measurements 2009/2010

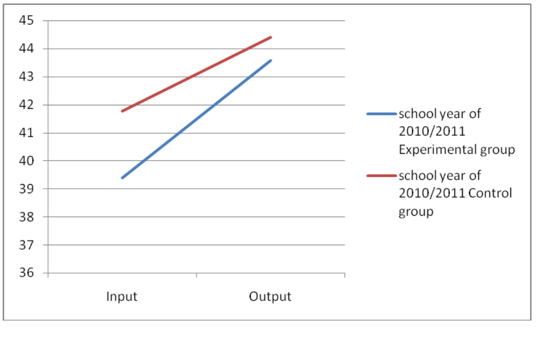

Diagram 5: Comparing measurements 2010/2011

On the basis of the diagrams the following can be stated:

- in each case the development of the GeoGebra experimental group showed a faster pace

- the GeoGebra group despite starting with a few % of drawback at the start of the school year, managed to catch up (in school year 2008/2009 it even exceeded the results of the control group as it can be seen in the school year's final measurements)
- The rate of development in the case of the control group was respectively 1; 2 and 2.5% compared to the conditions at the beginning of
- The rate of development in the case of the GeoGebra group was respectively 4.5; 3 and 4% compared to the conditions at the beginning of the year

Besides the above mentioned quantitative natured characterization, follow-up the Claus-questionnaire indicated that the inclusion of computers motivated the pupils in a very great proportion. According to one of the sayings of György Pólya, math teachers have to be like good merchants: they have to persuade the customer to buy the goods. [7] In my opinion this can be primarily achieved by influencing the pupils' affective factors. And if we have achieved with regards to the pupils that there is a certain motivational base (as a consequence of the pupil's attitude towards mathematics) on which we can build on, then with purposeful, planned development we will be able to affect the cognitive and psychomotoric factors as well.

The experimental period was characterized by a continual methodological control, which besides including many others were the following:

- The continual observation of 8/a and 8/b during the last 3 school years
- Claus-type questionnaire
- A pupil satisfaction questionnaire related to the usage of computer and the inclusion of GeoGebra
- The continuous oral questioning of pupils with regards to classroom experiences
- The collection, analysis of independent products (independently prepared GeoGebra work sheets, documents...)

3 Conclusion

The demonstrable advantages of Geogebra's usage among disadvantaged/multiple disadvantaged children were the following:

- Creating motivational base for pupils
- Helping the schemes' individual design (assimilation, accomodation)
- Facilitating the development of cognitive abilities, thinking process (crossing into the stage of formal operations)
- Improving affective factors (a more positive attitude towards mathematics, the usage of computers in classroom motivates pupils)
- The development of psychomotoric abilities

During the observations of the 3 school years I experienced the following: among pupils who were difficult to motivate happened that sitting in front of GeoGebra, they worked through most of the class attentively.

The most important result for me is that a pupil came to me in the break, and told me the following about a GeoGebra class:

"Professor, when we are going to play again?"

(Roland Rácz, a pupil from class 8/b)

Acknowledgements

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Report on Mathematics "Test Zero" at Budapest University of Technology and Economics in 2011

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Abstract. It is essential for higher education institutions to implement strategies or steps to improve the teaching methods used in mathematics sessions, so as to subsequently reduce the problem of non-completion. As one element of these steps freshmen admitted by the Budapest University of Technology and Economics (BME) are required to take a test, called "test zero" in mathematics since 2009. This paper pictures the story of mathematics "test zero". The 2011 results of all 3300 participants are presented and attention is called to typical mistakes. Some of the most important and interesting conclusions are highlighted.

Keywords: Teaching mathematics, assessment, admission system, higher education, high-school graduation.

Classification: B40; D60; D70.

1 Introduction and background

Mathematics is vital for engineers but its role is changing. In 2011 SEFI (European Society for Engineering Education) Mathematics Working Group (MWG) produced the first revision [1] to their own report on a mathematics curriculum for engineers [2]. In this 2011 revision report they say: *"Mathematical competence is the ability to recognize, use and apply mathematical concepts in relevant contexts and situations which certainly is the predominant goal of the mathematical education for engineers."*

In the 2002 report "Mathematics for the European Engineer, a Curriculum for the Twenty-first Century" [2] they noted that *"in increasingly more countries, there is concern over the deterioration in the mathematical ability of new entrants to engineering degree programs. However, in increasingly more countries there is concern over the deterioration in the mathematical ability of new entrants to engineering degree programs."*

Hungary is no exception to this experience. There are increasing concerns in Hungary about the skills and competencies of students entering higher education. A nationwide survey was conducted among university students and a study about mathematical preparedness was published in 2009 [3]. A lack of mathematical competencies and abilities has been identified as a factor resulting in non-completion of courses in Hungarian higher education institutions.

Admission points are based only on high school achievements in the Hungarian enrolment system. Points are calculated on the results of the final exams (graduation exams) in two high school subjects, one of which is typically mathematics for applicants for engineering programs. The maximum of total admission points is 480 including extra points. 400 points are based on the grades in the last two years of secondary school as well as/or the results of the graduation exams, while applicants may earn a maximum of 80 extra points for extra achievements. High school students are given options before they sign up for the exams: they may take the graduation exams at advanced or intermediate level in each subject. The advanced level exam is more difficult, covers more topics and has a written and an oral part. The intermediate mathematics exam only has one written part. It is more likely that one can earn better test scores at the intermediate level. Even though students are encouraged to choose advanced level exams by the possibility that they may earn an 40 extra points for each very few of them makes this choice.

Pilot programs for measuring the mathematics knowledge of freshmen when they start their studies were introduced at different institutions of higher education in many countries [4, 5] and also in Hungary [3, 6, 7] during the past few years. A nationwide assessment of freshmen was introduced in mathematics, physics and chemistry in 2009 according to a former decision of the Panel of Rectors and Presidents of Hungarian Technical Universities and Colleges. Papers about the distressing results were published [8].

Budapest University of Technology and Economics (BME) with its 23 000 students is a leading institution in engineering in Hungary. Lecturers and professors in mathematics of BME also have the impression that preparedness of freshmen is becoming worse and worse every year. The failure rate is increasing together with the proportion of dropouts or withdrawals. To maintain the necessary standards has become a challenge. This has created the need to identify why so many students struggle with mathematics. The idea of adding "test zero" to items of the course requirements of first semester mathematics courses was motivated by the fact that students are not required to take an admission exam and the university has no other information about the knowledge and the educational background of the student than his/her total number of admission points. A report on the results of 2010 test zero was formerly presented [6]. Some parts of that study are used in this paper.

2 Objectives and methods

The goals of "test zero" are

- to define clearly the prerequisites of first semester mathematics,
- to enforce students to refresh their former knowledge in mathematics,
- to test certain mathematical competencies,
- to give feedback to students at the very beginning of their studies,
- to give the opportunity to students who have failed to sign up for make up courses to improve their skills,
- to obtain data about the mathematics background of freshmen: to get information about how the total admission points and the test scores are correlated; to get information about how the level of the high school graduation exam (advanced or intermediate) in mathematics and test scores are correlated; to get a list of topics in which the students were less successful,
- to give feedback to secondary school teachers and educators.

Students took the test in the second week of the fall semester on 12th September 2011 [9]. The time for the test was 50 minutes, the paper-based test included 15 multiple choice questions with 5 possible answers of which exactly one was correct. Students had to write the code (A, B, C, D or E) of the correct answer into the answer box. Since the number of participants was so large, 6 different versions of the problem sheet were used with all similar problems.

Students were not allowed to use pocket calculators, formula books or formula sheets. One correct answer was worth 4 points, one wrong answer was worth -1 point and the student got no score (zero point) if he/she had left the answer box blank. The maximum test score was 60, the minimum test score was -15 points. Test was supposed to be successful if its score was above 24 points (40%).

To set up the items of the test we also studied

- Hungarian high-school curricula,
- placement tests of several universities including ones in or outside Hungary,
- the PISA assessments,
- admission exams of the former Hungarian admission system,
- competency based curriculum recommended by the European Society for Engineering Education (SEFI) Mathematics Working Group [1, 2].

Instructions and a sample test were available on the website of the Institute. A 3-day make up course was offered to students during the registration week. The participants were first year students of BME, among them bigger groups of students of the following faculties:

- Faculty of Civil Engineering (380 students)
- Faculty of Mechanical Engineering (699 students)
- Faculty of Chemical and Bioengineering (291 students)
- Faculty of Transportation Engineering and Vehicle Engineering (346 students)
- Faculty of Electrical Engineering and Informatics (938 students)
- Faculty of Economic and Social Sciences (592 students)
- Faculty of Natural Sciences (98 students)

The number of all participants in mathematics “test zero” was 3344 in September 2011. The answer codes were recorded into an Excel file by student workers in one night. The lists with the total scores of all 3344 participants were available on the internet in less than 24 hours after the test was taken [9].

3 Results

3.1 Total admission points and test results

Figure 1 shows the distribution of test scores for all participants.

In order to gain an overall view of how the admission points and test scores are related we show Figure 2. On Figure 2 every point represents one student, identical points may appear. A major part of the box is almost uniformly spread by points. Having a student with excellent admission points we can hardly suggest what we

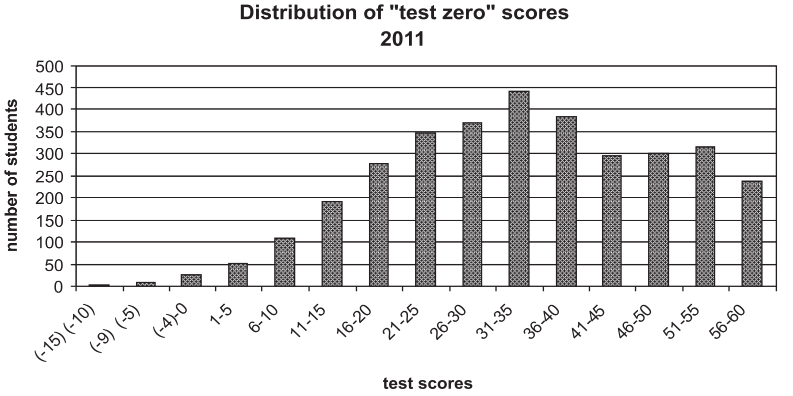

Figure 1: Distribution of test scores for all participants

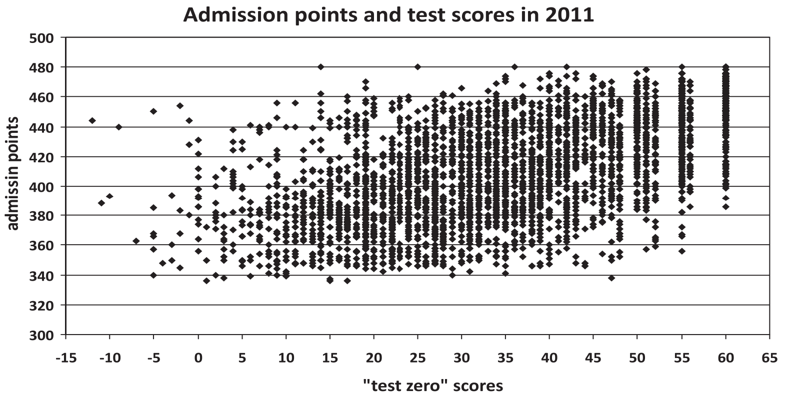

Figure 2: Admission points and test scores for all participants in 2011

can expect from him/her in mathematics and knowing that the student has a bad test score we can not conclude that he/she belongs to the lower group with respect to admission points. But if the student has a good test score (above 75%) it is more likely that he/she will have good admission points (above 400). Based on the figures one may assume that the correlation between test results and admission points is not very strong. Admission points give us insufficient information about the knowledge of the student.

Even though lots of students have excellent admission points, students with non-traditional mathematics backgrounds are at a risk of struggling with problems of “test zero” and consequently with the other requirements of college-level mathematics courses, due to possible gaps in their mathematical knowledge.

3.2 The level of high school graduation exam in mathematics and test results

Knowing that high school students may make a choice on the level of their graduation exam in every subject it is not surprising that very few of them choose the advanced level in mathematics, which is assumed to be difficult. Even though that the university curriculum in mathematics was prepared assuming that students are familiar with the concepts, facts and formulas covered by advanced level graduation exam only 32.7% of the freshmen of BME involved in "test zero" arrived with an advanced level mathematics graduation exam in 2011.

	Advanced	Intermediate
Proportion of participants with this level of graduation exam	32.7	77.3
Average "test zero" result (%)	72.5	49.2

Table 1: Total admission points and average scores in different groups of students

It is important to emphasize that it is much more likely that a student with an advanced level graduation exam meets the requirements (40%) in "test zero" of BME. Table 1 and Figure 3 demonstrate this fact.

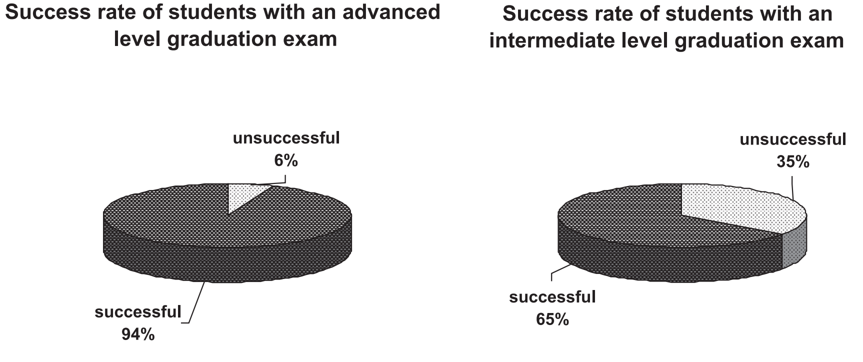

Figure 3: Result of students with different levels of graduation exam

3.3 Topics that seemed to be difficult for students

There were problems in which the success rate was lower than in others. Among these problem areas we found the ones on algebraic skills, trigonometry, word problems, inequalities, geometry, and vector algebra.

In the following section some of these problems will be presented.

Problem 1. Algebraic skills.

Find the value of $\dfrac{\lg 9 + 2\lg 2}{\lg 6}$.

Number of answers		
A	132	16%
B	153	18%
C	12	1%
D	7	1%
E	287	35%
Blank	240	29%
Total	831	100%

Table 2: Distribution of answers in problem 1

(A) 6
(B) $\lg 6$
(C) $1/6$
(D) 1
(E) 2

831 students were asked to answer this question. The distribution of the answers is shown in Table 2.

The ratio of the correct answers (E) was only 35%. Though the ability to use algebraic formulas correctly should have already been developed by the time of finishing secondary education, more than one third of the students proved to be willing to accept false answers. One may notice that students chose the distractors (A) and (B) with an extremely high rate.

Problem 2. Vectors, scalar product.

Given the vectors $\mathbf{a}(-1;1)$ and $\mathbf{b}(2;4)$ find the cosine of their angle.

(A) $-\frac{1}{\sqrt{2}}$
(B) $\frac{1}{\sqrt{2}}$
(C) $-\frac{1}{\sqrt{5}}$
(D) $\frac{1}{\sqrt{5}}$
(E) none of these

831 students were asked to answer this question. The distribution of the answers is shown in Table 3.

The ratio of the correct answers (D) was 56%. The relatively high rate of correct answers and the fact that only 27% of the students left the answer box blank this year is likely due to the fact that exactly the same problem had been asked formerly in each and every problem sheet that were all available for students in the internet. Good news is that the rate of correct answers is increasing with time. The university curriculum was prepared assuming that students are familiar with the concepts and formulas of basic vector algebra like dot product of vectors. Lecturers and instruc-

Number of answers		
A	10	1%
B	46	6%
C	23	3%
D	465	56%
E	35	4%
Blank	252	30%
Total	831	100%

Table 3: Distribution of answers in problem 2

tors of BME have to learn that the ability of students to identify and use formulas like the dot product correctly is limited though it is a basic requirement in other technical subjects also.

4 Conclusions

The study presents informative findings and results from a recently conducted survey of mathematics knowledge of BME students when they enter higher education.

Conclusions that were obtained from the analysis of the test results in 2011 are pretty much the same as those of 2010 [10], namely:

1. Admission points and test scores are hardly related. This means that admission points can not be used to predict the performance of the student.
2. The rate of success in the test is closely related to the level of the high school graduation exam in mathematics. Students who arrive with an advanced level exam are more likely to meet the 40% result, the required standard in "test zero".
3. Several students enter higher education in the engineering programs of BME with insufficient knowledge of mathematics.
4. The topics that seemed to be more difficult for students or in which the success rate is lower are identified: algebraic skills, including ability to use identities for logarithms, trigonometry, geometry, word problems, inequalities, vector algebra.
5. Diagnostic testing (and the followup support) can help student retention and hence save time and money for both students and university.

Seeing these results, BME decided to open extra elementary bridge-gaping mathematics courses that offer the additional benefit of aiding the students in their transition into higher education. In the fall semester of 2011 1074 students registered for such an extra course.

With the current eclectic mix in the background and abilities of current students, all programs in which mathematics is an obligatory subject should use this kind of diagnostic testing on entry. It is essential for higher education institutions to implement strategies or steps to improve the teaching methods used in mathematics sessions, so as to subsequently reduce the problem of non-completion. The provision of mathematics support through extra elementary courses seems to work successfully at BME.

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How we teach Mathematics?

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Abstract. The teaching of mathematics is important not only for the application of the acquired knowledge, but for the development of the logical and rational way of thinking of the students of economical studies. Mathematics would be the most appropriate school subject to develop certain skills and abilities such as: associating, selecting, spotting the essence, abstract thinking, logical thinking, clearness of thinking, sense of locality, creativity, self control. Mathematics bears a particular danger which tempts the teacher to concentrate his attention exclusively on the cognitive aims of teaching, such as: rational argumentation, formative skills (algorithms, calculations), mathematical modelling. With the help of a questionnaire I made a survey in which I viewed the affective (emotive) component: the general interest, the perseverance, the attitude towards the content of learning material, predisposition, inclination to learning, motivation, activity, conviction. The survey was carried out among the first year students in economy and it views the methods they met in practice while learning.

Keywords: secondary school teaching, mathematic abilities, basic knowledge, problem solving, motivation, economic sciences.

Classification: C20; C70.

1 Introduction

The secondary school teaching plays an important role in the development of the problem solving skills and of the knowledge needed to acquire these skills. It also has a great influence upon the ability of acquiring specific knowledge required in the high education (at the university) of subjects at the Faculty of Economics such as economic mathematics, economic statistics, calculation of probability, accounting, etc. One of the main features of the Romanian educational system is that student can study Mathematics at different levels in highschool. For example, a student in Mathematics-Computer Programming Department (called Profil Real) of a National College may even take a Baccalaureate examination of high mathematical knowledge as the programme includes: differential equations, probability theory, the basics of linear algebra, complex numbers, etc. There are many mathematical subjects in the curriculum of BA/BSc, MA/MSc and PhD degree programmes of faculties of economics. Just like Physics, Economics needs the synchronic use of different subjects and domains, including all the branches of mathematics. (Kánnai, Pintér, Tasnádi [2010]) In the afore mentioned work there is a short presentation of the types of mathematical knowledge needed in economics. The following basic subjects in mathematics are part of the curriculum of Economics department at our university: financial mathematics, probability theory, financial statistics each throughout a semester. There is no doubt that students need a solid basic knowledge acquired in high school in order to become a good professional or to get a PhD degree.

2 The emotional aims of teaching Mathematics

"To teach effectively, a teacher must develop a feeling for his subject; he cannot make his students sense its vitality if he does not sense it himself. He cannot share his enthusiasm when he has no enthusiasm to share. How he makes his point may be as

important as the point he makes; he must personally feel it to be important." George Pólya, Mathematical discovery (New York, 1981).

Mathematics bears a particular danger which tempts the teacher to concentrate his attention exclusively on cognitive aims of teaching, such as: rational argumentation, formative skills (algorithms, calculations), mathematical modelling. The use of Mathematics means creativity because all these can be easily turned into mathematical activities, applications and can be checked, making possible the evaluation of effectiveness of teaching.

Since the 70s there has been a growing research of the emotional side of learning and teaching, including: extended attachment to the learned contents being taught, affinity to learning, motivation, general interest, involvement, conviction. According to Erich Ch. Wittmann (Ambrus [2004]), the affective aims of teaching Mathematics are the following:

1. Developing positive attitude and interest towards Mathematics and the joy stired by it
2. Individual student work done with self-confidence
3. Ability for group work done with pleasure. Giving help and learning from the others
4. Finding intellectual inspiration and satisfaction in Mathematics
5. Determined, enthusiastic, concentrate work. Thrive for comprehension
6. Feeling of joy and pride at the end of a successful work in Mathematics
7. Recognizing the relevancy and harmony of Mathematics, understanding it
8. Appreciating the clearness of mathematical concepts and appreciating its relative certainty
9. Appreciating the beauty of mathematical object, theory, consideration

2.1 Research

The research is based on 2 parts. In the first part I made a survey based on the questionnaire of Claus (Ambrus [2004]). I enlarged it a little bit in order to analyse the affective factors: the general interest, long termed attachment, attitude towards the learning content, predisposition for learning, motivation, activity, conviction, as well as the change of all these concepts in time. I was interested in their negative and positive feelings towards Mathematics, their experiences and attitudes. I made the research last year on 70 freshmen in economical studies at the Partium Christian University of Oradea. In the second part I made the same survey this year with some more 90 freshmen in Economics, looking into the following questions:

- what kind of methods and instruments did they meet in high school;
- how did they learn Mathematics from the point of view formality and application;
- what did their teacher find to be most important;
- where would they place Mathematics in the list of subjects on a scale of preferences;
- what is their opinion about their own Mathematical abilities, skills and work.

In these questionnaires they had to answer more than 120 questions. They filled in the first questionnaire of 30 questions under my survey while they answered the rest of 90 questions on the net and sent the answers to me.

2.2 My hypotheses

1. The teaching of Mathematics is more and more informative than formative, despite the curriculum which regards the development of skills and competences.
2. We, the teachers do not use a varied range of alternative methods. The traditional teaching bears the emphasis on the automatic, routine applications rather than on understanding the concepts and on individual thinking.
3. We offer the students the solutions and we do not let them enough time to discover and find by themselves the solutions.
4. During the maths classes the use of multimedia devices, the PC, the internet is the least present. Teachers use them very rarely.
5. The interest towards Mathematics and towards sciences in general has decreased a lot recently.
6. From the students' point of view, Maths classes are boring and abstract.
7. The interdisciplinary teaching is not realised.
8. In the primary and secondary school, Maths is the students' favourite subject, while during high school it is no longer.

2.3 The results of the survey

80% of students have graduated high school this year, while 20% graduated school earlier. From the point of view of gender: 54.44% of them are women and 45.55% are men. Regarding their origin, they come from six different counties and different schools.

The first thing I discovered after doing the survey is that 62% of the students have graduated Human Studies at high school, where they had had just 1 or 2 math classes a week and only 38% of them had studied analysis. That percentage of 62 was possible due to the fact that while there is no entrance examination (acceptance to a Faculty is 80% based on the average mark of the students' studies throughout the 4 years at high school and 20% is based on the Baccalaureate results), so the maths test is not compulsory on the Baccalaureate examination. This year the situation is the reversed: 63% of students have graduated Sciences and just 37% Human Studies in high school. So there might be a great difference between the generations.

In *"How to solve it - A new aspect of mathematical method"*, George Pólya writes about an American survey which shows that mathematics is the least favourite among all school subjects. (Pólya [2000]). That proves to be just partly true. Last year the answer to the question whether Maths was among their favourite school subjects was as it follows: 58.20% enlisted it among their favourites during their study at secondary school, while just 31.34% chose Maths to be among their favourite subjects at high school and finally only 10.44% at university. The rest of 37.31% have never liked mathematics.

This years freshmen have ranked mathematics on the second place after foreign languages. Natural sciences and art follow it next. Unfortunately Biology, Chemistry and Physics are the last in the line.

If they were to choose the subjects to learn, only 16.41% of them would choose mathematics, 79.10% wouldn't while the rest of 4.47% may choose it. If they had the time 4.47% of students would take up mathematics as a hobby, while 26.86% wouldn't and 68.65% like to solve puzzles, sudoku, etc.

The rate of students who are afraid of making a mistake and get mocked at is as it follows: 22.38% in secondary school, 26.86% in high school and 28.35% at the university. The rest of 49.25% have never been afraid of that. (50.74% of students have always been afraid of making mistakes at calculations.)

In order to make students love mathematics the teacher plays an important part in the secondary school, according to 80.59%, in high school according to 79.10% and at university, according to 76.11% while the rest of 5.97% thinks it does not depend on the teacher. If they were to choose their job, 25.37% would like to teach mathematics in secondary school, 5.97% would teach it in high school, 2.98% at university and the rest of 70.14% wouldn't like to be teacher.

71.64% had a lot of satisfaction and felt well at maths classes in the secondary school, 61.68% in high school, 29.85% at university and 10.44% said that they have never had such a feeling, the feeling of success at maths classes. 59.70% of students prefer to do 5 routine exercises rather than one difficult in secondary school, 53.73% in high school and only 38.80% at university!

79.10% of students think that mathematics is different from the other subjects, while 20.89% think it is not. 22.38% of students think that mathematics is such an abstract and difficult subject that one can barely understand it in the secondary school, 34.32% in high school and 73.13% at university. 16.41% of the students never had such problems.

They used the same solving method and practiced it throughout several exercises: almost always - state 35.70%, most of the times -39%, just on few classes 23% and 2.3% say they have never done that. We learn through practice and through solving problems: almost all the time, think 60% of them, on most classes, think 32.22%, just on several classes, say 6.66% and 1% think they never. 52.30% say that they have always listened to the teacher in the classes, 42% on most occasions, 5.7% just sometimes and 1.11% never.

According to 56.66% there was never noise and disorder during the math classes, according to 33.33% there was only on several occasions, 5.55% think there was most of the time and 4.44% say there was almost all the time.

We have solved such types of exercises that help us in our everyday life: 45.55% say they have never done that, 28.88% say they have done that during some classes, 17.77% on most classes and 7.77% say they did that almost all the time.

We solved the difficult exercises gradually after having solved easier ones: 1.12% say they have never done that, 9% say they have done that on several classes, 41.57% on most classes and 48.31% say they have always done like that. We can influence the course of the class: 12.22% think they can never, 37.77% think they can during several classes, 38.88% think it is most often the case and 11.11% almost always.

We can choose the exercise, the method and we can have our own ideas: never, according to 13%, sometimes, according to 36.5%, on most occasions, is the opinion of 38.8% and 11.5% say they can always do that.

Our teacher gave us precise instructions: sometimes- according to 12.5%, on most classes, according to 43.82% and 43.82% say it was always the case. The teacher helps those who need it: on some classes, according to 17%, on most classes, say 30.7% and 52.3% think it happens almost always. The teacher summarized the details and the results: never, say 2.25%, just on several times, assert 13.5% , most of the classes say 42.7% and 41.6% say the teacher does that almost always. The teacher loved the subject he/she was teaching: 90.9% of the students think that their teachers loved mathematics.

The teacher used frontal way of teaching: never, say 3.33% , just on several occasions, according to 14.44%, on most classes, remember 57.77% and 24.44% say that it happened almost all the time.

There is a talent in mathematics: is the opinion of 56.8% of students, while 43.2% think it does not exist. I often talk about the experiences and happenings of the maths classes: is never the case for 15.11% of students, it sometimes happens in the case of 48.8% , and often for 27.9% while 8.13% say they always do that.

I look forward to the maths class: it is never true for 8.88%, it is sometimes true for 48.88% of students and in most cases for 36.66% while 5.55% say they are always excited about the math classes. I'm happy to learn this subject: 4.44% think the opposite, 20% think it is not so good, 57.77% think it is a good thing and 17.77% say they are very happy to learn mathematics. I easily learn Mathematics: say 60% of students and 40% say they do not. I would like to have more maths classes: say 5.55% of students, 17.77% would agree with that, 57.77% wouldn't really enjoy it and 18.88% wouldn't like it at all.

I think this subject is very useful for us: 7.86% say the opposite, 24.71% think it can be sometimes, 50.56% think it is in most of the cases and 16.85% think it is almost always useful. It would be nice to use mathematics as a grownup in my job: 12.22% say the opposite, 50% say they wouldn't really like that, 31.11% say they would like that and 6.66% say they would particularly like that.

I listen to the teacher when he explains: most of the time, say 55.05%, 38.2% say they often do, 4.5% sometimes do and 2.24% almost never do that. I ask the teacher questions: most of the time in the case of 7.9% of students, often do that 29.2% of the students, 60.7% sometimes do and 2.2% never.

I copy the method from the blackboard or from the textbook: 27% of students say they do that most of the time, often 52.8%, sometimes do 14.6% and never 5.6%. I work and think individually: most of the time 19.3%, often do that 28.4% and sometimes 45.5% while 6.8% never do. I share my thoughts and ideas with my mates and friends: in most of the cases, say 21.3% of the students, often, is the answer of 49.5% of students and sometimes for 27% while 2.2% say they never do that. I learn by heart the rules and features: most often, say 7.9%, 59.6% say they often do that and sometimes just 28% while 4.5% of them never do that.

We solve verisimilar exercises: most often, is the case of 12.65%, 54% think they often do that and 25.3% just sometimes while 8.05% think they never do. We analyze and talk over our mistakes: according to 38.2% of the students that happens most of the times, they often do think 50.6%, sometimes do 10.1% and almost never 1.1%.

3 Conclusion

About the experiences of maths classes and about the practiced methods:

If they could decide, just 16.41% of learners would take up mathematics. Most of them had the satisfaction and the feeling of success, though maths classes cause stress when writing tests or being at the blackboard. Many of them are afraid of making mistakes at the calculations (50.74%), still in most classes they analyze their mistakes in order to learn from them, according to 90%.

Teachers use in the class almost exclusively the frontal teaching: 80% of learners have met just the frontal teaching method during the 12 years of attending school, they have not met any other methods in learning mathematics, they have not used in class computers or internet. This fact underlines the hypothesis that during the maths classes the use of multimedia devices are the least often used and the computers, the internet are never or just very rarely used and the teachers do not use a variety of methods. The traditional teaching methods are based on the mechanical applications and routines rather than on individual thinking and understanding.

I copied the procedure from the board or from the student book, say 80% of learners. *I learned by heart the rules and features*, say 68% of them. That is the proof of my hypothesis that there is still a lot of formality in the teaching, to the detriment of the development of creative thinking and of competence. Developing knowledge based on abilities has not come yet.

They learnt by solving problems, using the same procedure through several exercises, starting with the least difficult one and continuing gradually with more and more difficult ones. According to 75% of the learners they solved such tasks that are related to their everyday life (that is interesting because the textbooks contain very few exercises that may refer in content or in formulation to reality). Unfortunately they would prefer to solve usual or routine like tasks, of which they already know the mathematical formula rather than another problem which needs thinking. 70% of learners share their thoughts and ideas with their mates. Half of the learners think they were able to influence the course of the math class.

Analysing the relation between the individual work, asking questions and working in his or her own style, the strong bound among these components is revealed, proved by the result of a Chi-squared probe which is 10.638, the Yule-coefficient is $Y=0.6385$, which shows a quite strong relation. So, the individual work is indispensable for individual thinking and implies it in the same time, making the learner to ask himself some questions and to do individual mathematical activities.

95% of learners listened to the teacher when the teacher explained the lesson. Noise and disorder are not the characteristics of the maths classes.

88% of the maths teachers are precise, according to the learners, they gave exact instructions during the classes and they helped those who needed it, summarized the details and the results and 90% of the students think that their teachers liked mathematics, the subject they were teaching.

(If we use only traditional methods, the role of the teacher remains the same: he is the leader who helps directly the learner to find the solution. Quoting George Pólya: *"It is important what the teacher says in class but it is much more important what the learner thinks. The idea must arise in the head of the learner - the teacher should only be a helper...the guiding idea is that the learners should find themselves the solutions and discover all they can in the given situations."*)

80% of the students think that the teacher plays an important part in making the learners love mathematics, no matter the level, primary school or university. It is surprising that 75% of students wouldn't be a maths teacher.

More than half of the students have mathematical abilities or talent. 37% of them usually talk about maths to the people around them and 40% are looking forward to the maths class. 15.57% use to work on mathematics in their free time.

About the utility of mathematics in their future life as grownups,the way they will use it: 38% would like to use it at work and 67% think that this school subject is useful for them.

The learners should see more and more practical approach. Modelling and the practical application of concepts are part of the curriculum but the mass of the content and the time given to it do not allow learners to deeply sink into an application.

We should make our methods more varied. The aim is to use such active- interactive learning forms and such methods and teaching aids which would help the teacher make his classes more exciting, more interesting, guiding them towards some new experiences by working individually as well as together. Thus the teacher can help them develop their problem solving ability and can pay attention to certain learners' individual needs making them an active part of the lesson. Experiences all over the world show that the guided discovery is practicable at all levels of teaching mathematics, in teaching the new lesson as well as in revising and solving problems. The maths teacher should revise his teaching methods in order to become a partner in learning to his students. According to George Pólya, the teacher is a businessman who wants to sell mathematics to young people whose interests are totally different. He should attract them in his shop with such exercises that are taken from everyday life and which would match their interests and which finally would naturally stimulate them to work individually. Our question should be of the sort that it raise the shopping interest and make them venture into an intellectual journey. The best motivation can be the interesting, exciting problems which are close to the students and which bring true joy by solving them. It is also important for the learners to ask questions and formulate problems because thus they would be more eager to think about what they formulated themselves.

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Commensalism between board games and teaching maths

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Abstract. If you ask in Slowakia someone to name some board games, probably you will hear titles like "Človeče, nehnevaj sa" (Pachisi), Monopoly, "Dostihy a sázky" (no english distribution or name known), Risk and a few others. But nowadays a new form of board game named "Euro" is becoming very popular. Euro games are well suited to use in schools. They require and utilize skills, they stimulate ways of thinking and they don't last too long. The board games bring cooperative learning, social interaction, and what never should be forgotten, fun into your lessons. In the article we show mathematic skills evolved by playing games, we make proposals, how to use them on regular math lessons.

Keywords: Board games, mathematics, teaching maths, skills, arithmetic, algebra, geometry, strategy, tactics.

Classification: U60.

1 What are games good for?

Every year a thousand or more board games are published around the world. The authors (in board games business named designers) are no more nameless. A lot of them are living from designing games. And the games have more and more benefits to pedagogical focuses. Sometimes the foci are hidden, sometimes they are direct. But if you search for some subject, you can really find games with mechanics, milieu, knowledge etc. in the wanted area.

In Germany in the years 2003-2004 researchers realized a project named "Spiel frdert die Schule" (Games assisting in schools). Afterwards in 2006 there was made a half year evaluation of the conclusions of this project. The evaluation confirmed 7 reasons for using games in schools. Playing games develops [1]:

1. intelligence,
2. personality development,
3. social behavior,
4. motion ability,
5. power of concentration,
6. linguistic and/or language development,
7. creativity.

As teachers of each subject are developing not only the knowledge, but the whole personality of their students and pupils, all of these conclusions are important and snatching.

Pritchard [2] is describing similar efforts: "The most useful skills learned through playing games are some of the most valuable the school system can provide, namely the interpersonal and intrapersonal skills involved with respectable and meaningful person-to-person interaction. The beauty of board games is that they are played by a group of people. Whether the group is all friends, all strangers, or a mix of the two, the game is the focus of their social interaction for the playing time, and in that time individual players will sometimes benefit and sometimes disadvantage other players. All of these interactions, combined with the fact that in a game there can (usually) be only one winner, mean that players learn—or should learn—to treat each other courteously, to behave in an honorable fashion, and to be good sportspersons. (Some games are cooperative, with players competing against the game itself, and these are excellent for teaching the dynamics of teamwork)."

2 Mathematics behind, in and overlapping the games

The games have their designers, which designed a good working structure. They try to minimize a straightforward winning strategy. The game should give more possibilities and decisions to any player on his turn. There is much more maths than the player can see behind. A good example can be our interview with Friedeman Friese while playing his Factory Manger. I asked him, why are all the robots always 4 Electro more expensive, but the last one is 3 Electro more expensive. He told something in the further given way: I counted the efforts and the price of this tile. With a view to the time it is coming to play, there was no possibility to let the increase in value to be constant. So I counted the value of each of the tiles over... That is the mathematics behind, the mathematics which should be done before publishing.

But there is mathematics in the game, which we do see and maths overlapping the game which we do, but pupils do not see. Both of them give us the possibility reach our pupils and make maths lessons more fun, more interactive, more unbound and more remarkable. We could publish some lists of games for kids of every age. In [3] you can find a list of games for first grade children. In [4] the list is spread for children from 10 years and in [5] we are concentrated on geometry in all her kinds. In this article let us see how much maths in the next 3 games is hidden. We did choose different games – different in the focus of age, time, and also maths.

2.1 Picomino (Heck Meck z zizalek)

Picomino is a game for children from 8 years age. In the box of the game there are 16 dominoes and 8 dice (cubes). On each dice we have numbers of dots from 1 to 5 and on one side there is a little worm there. The dominoes have really two halves as usual but with strange paintings. On the one halve we see numbers from 21 to 36, only one of them on each tile. On the other halve we see from one to four little worms. With a higher number more worms, with a lower one less worms. The clue of each player is to collect the most worms. A worm is 5 dots worth. On his turn a player rolls all dice, sets all dice with one symbol aside and is allowed to roll the left over dice. After each rolling he must set dice with a new symbol aside. If he has after this on the dice set aside at least one worm and a sum which is bigger or equal to the number on one of the dominoes, he can end his turn and take the domino. In a special case the player can steal a domino from another player.

This game is good for training counting. As written in [3] "the children slowly change from counting dot by dot to recognizing the schematic representation of numbers and their counting as whole." The pupils in the second or third grade have sometimes problems with counting without fingers. Here on the dice are dots, which the child can see and they can represent the otherwise used fingers. Counting becomes quicker and using fingers will be lesser frequent. This is the mathematics all can see in the game. But there is something else. From counting three fives to multiplication three times five is only one little mental step. That is not all. If a player rolls the dice, he has to choose dice with which symbol he will set aside. Is it better to take two dice wit 3 dots each? Or should he take one dice with 5 dots? Or better one dice with only 1 dot? As the player has only 8 dice, he must also count if he can make at least the lowest number on the domino. And the next step – he can say stop, but he can roll once more. Is it safe, to roll three dice if there are only 2 symbols not rolled yet? So we can find a first contact to probability.

If you use Picomino in your class, one copy is for 2 – 8 players, so 4 copies should be enough for the whole class. The game lasts from 5 to 15 minutes. This depends on the speed of counting. But let us say: would your pupils do so much written exercises in the time they do while playing the game? You can use the last 10 minutes of your lesson for playing. You can also make a tournament on only one whole lesson. All the time invested into the game comes back from the progress of your pupils.

2.2 Carcassonne

Let us go to children from the third (better fourth) grade. Carcassonne is nowadays a well known game all over the world. There were millions of copies sold and the game has a lot of expansions. So you can expect that at least one or more pupils will know the rules. The game consists of square tiles and meeples. There are cities, roads, monasteries and meadows on the tiles. A player on his turn picks a tile and then he must place it. He must connect the tile with one or more of his edges to the existing land consisting by further laid tiles. Placing is only possible if all the land, city, road continues on the connected edges. Then the player may place on the tile one of his free meeples. As a last action the player scores for all closed cities, roads or monasteries. The points belong to that player, who's meeple stands on the closed land art and the meeple goes back to him.

After reading that rules we could only say: where is another mats than counting points? We will show you. There is much more maths as anybody would expect.

- Arithmetic and algebra – Is it better to set the meeple into a city try to connect it later with another city and win the points from it (the active player +4 points, the opponent loses his points from the city)? Or should you lay the meeple to the meadows, which you do not know, how many cities will be delivered by (unknown how much times 3 points, but the meeple is until the end of game unaviable and you score the same amount of points for the city.);
- Using algorithm – each game rule is a description of turns - an algorithm;
- Strategy – from the first turn on you can play on cities, on monasteries, on taking over the opponents cities... You have a plan and this plan is nothing else than strategy;
- Tactics – In the moment you draw a land tile, you have to decide, how you will react to the opponents turn, what is the best choice with this tile;
- Memory – is very important in tournaments. The player knows about possible tiles, he is waiting for some special tile and he should remember how many of them in the game are how many were already used...;
- Geometry – geometry and combinatorics are part of locality – it means decisions how to place a tile, how to rotate it and where to place it. Some pupils will "see" the right place at once, others will need some time, and others

will need to try. From play to play they all will see more and more without manipulation. It means their mental ability to solve such a problem increases.

The game is recommended for 2 – 5 players 8 years and older. It means you need up to 6 games in your classroom. A game can be played in 30 minutes, but you can also use chess clocks with a preset amount of time for each player. If you want to use the game in your lessons, let at first the pupils only play and understand the rules. Afterwards you can use the games to count the maximum of points from cities, roads, meadows... or only note the score for each player in many games and then use this set in statistics lessons. The game is well suited for making tournaments. If choosing this way, we recommend playing two player games, as it is usual at the World Championship.

2.3 The Settlers of Catan (Osadníci z Katanu)

The Settlers of Catan is a game for children from 10 years. With 45 – 90 minutes it is the longest lasting game in this article. You cannot play it on regular lessons and the rules are also longer. If you are going to a school trip, with games you get a lot of program and if you have many of them, each child can be satisfied. And the good thing is, they learn something without knowing about learning.

The Settlers of Catan is a game with modular board. The modularity provide for always new feeling new appeal and high motivation to play. The player set two little villages and two roads each on the board. Then they collect resources to build new roads and villages and rebuild them to cities. A player on his turn rolls two dice and the sum of them will determine the positions giving resources and with the position the sort is also given. The player gets victory points for each village or city and he can get more points if buying development cards or having the longest road. All you do is paid in resources: wood, brick, wheat, sheep or ore. The player who reached in his turn 10 victory points wins. For exact rules in English see [6].

In this game we can find the probability on dice roll result. We can also count what is better: a position with the numbers 5, 10, 8, or a position with numbers 6, 9, 5? Where a player has better chance to get a resource: on a 9 or on a 5?

The game is full of planning (strategy) and making decisions and also trade (tactics). If you do not have a resource, you can try to trade with other players. But which prize for a resource is good for both sides? What does it mean good? Will a player give me his only one brick? What should I offer him? The brick means a village and 1 victory point for me. What is one victory point worth for him?

3 Conclusion

We tried to show how much maths is in board games. They can be used in the lessons; we only should look about some conditions like:

- educativ character
- theme
- fun factor
- how long will the rules last
- how much player we have
- how long it lasts to prepare and then store the game
- what do we expect from playing the game

What we really should have in our schools is a game collection, which can be used in classrooms. The collection can be divided by theme into maths games, geography games, social games, etc. But they can also build a game library, where children can borrow games as books, where games can be played. We agree with Pritchard [2] that: "Games can be used in a variety of ways to teach, reinforce, and assess, not to mention (perhaps most importantly) to have fun. A part of being able to use games appropriately at school is picking games that will suit the intended use of the games collection. Games can, and with familiarity will, be used in classrooms to support or extend teaching foci, but a school-based game collection can sometimes best support extra-curricula clubs and activities such as a school games club, board game nights, usable board game libraries, and the like."

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Measuring the efficiency of e-learning programs

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Abstract. In October 2011 I collected data in the American International School of Budapest on the "WebAssign" - the homework solver online program they use. I asked the students to fill out a survey about their habits, experiences and opinion about WebAssign, and had several structured interviews with teachers about the program. In the paper I introduce the observations I made, and part of the results of the survey that I have come into so far, while also mentioning some methodological questions to consider when measuring the efficiency of such programs.

Keywords: e-learning, WebAssign, efficiency of e-learning, learning management systems.

Classification: C70, R20, U70; D60, Q70.

1 Introduction

In the last decade, with the improvement of Information and Communication Technologies and the Internet, all sorts of electronics-based supporting systems have gathered ground in education. Education supported by computers (e-learning from now on) was introduced by the educational institutes as an opportunity created by modern technology to improve the quality of teaching.[1] There was also an intention to increase the efficiency of education, and to reach out to a new circle of students. The necessity of e-learning programs is no longer disputed, given the strongly increased number of students in higher education, which is not matched by a similar expansion of human resources.

The question still remains though, whether these programs are capable to provide the same quality in education as learning in small groups, through personal attendance. However important it would be to know the answer, today there are no systematic surveys of efficiency in Hungary, and even the occasional measuring of effectiveness is still quite rare. Reasons are many, for example the problems of methodology, which will be discussed later.

This is so in spite of the fact that there are numerous elaborated evaluative systems in use around the world, and even in Hungary (see *e.g.* [1]). In 1998 Kárpáti and colleagues made a research on electronic teaching tools used in Hungary with the participation of teachers using these tools.

2 Measuring efficiency: aspects to consider and experiences with WebAssign

2.1 Types of e-learning programs

Today many forms of e-learning programs or platforms are used by different educational institutes, among them there are many big, complex systems, which can be customized by inserting all kinds of modules (functions). They cannot be explicitly

[1] E-learning is of course present at every stage of adult education, but the issue of this article is to discuss the experiences of systems used by educational institutes.

differentiated, for their available functions are more or less the same. While we cannot state, that one program is only for administrative, the other is for communication purpose, and others are for defined educational tasks only, it is still common in everyday practice that one function stands out, and the others retreat to the background. Therefore the expansion of program functions does not lead to increasing simplicity; it is more common that teachers and students of an institute uses three, four or even more programs, but only certain functions of each. Even if this phenomenon has no direct effect on learning the curriculum, it can still influence the general efficiency of education, so it is worth paying some attention to it.

There are basically two types of programs and interfaces (softwares, web pages, etc.) developed only and explicitly to teach mathematics in use, the goal of which is to pass over, deepen, drill and verify knowledge:

1. Programs with highly interactive interface for demonstration (for example GeoGebra).
2. Programs for solving problems and practicing (WeBWorK, Webassign).

These programs play completely different roles, even those with the same basic functions. Therefore there is no point in comparing the Geogebra with the WebAssign for instance. The Geogebra is a very useful demonstrative device of living classes; it helps spectacularly display certain issues or phenomena, while WebAssign is for practicing at home. These programs are usually complementers of traditional education and as such are tools of a "blended learning" method. This means that they are not really replacements of the personal connection between teacher and student but of the "paper-and-pencil" practicing, or blackboard-and-chalk demonstrating, learning methods [2].

Black Board, Neptun, Moodle, Coospace, and many other complex programs, platforms, the so called Learning Management Systems were developed and (at least in the Hungarian higher education) are mostly used to simplify the (institutional or a teacher's) administrative tasks. One can manage results, grades and attendance registers with those, and they provide an opportunity of communication between teacher and student, directly assisting education. Materials of lectures and study-aids can be uploaded, home works can be sent in, and even tests can be filled out (for example in the Moodle) [4]. However, these programs are not made for teaching mathematics, so their efficiency in this field is way behind of those created for such purpose (see WeBWorK or WebAssign), though they can be (and are) further improved. Even the integration of aforesaid types of programs with more interactions and practicing interface is theoretically possible into these types of systems.

2.2 Problems of measuring efficiency

When measuring the efficiency of educational programs, the first and foremost question is defining efficiency itself. What is measuring based on, what aspects do we consider, what do we compare to? What are the expectations, who will be the users (high school or university students), what is the level of mathematics they wish to learn?

Practical possibilities set a strict limitation to these theoretical considerations. It is obviously impossible to measure how efficiently can the same student learn and understand the same curriculum with different methods. Therefore the accurate methodology of the measuring gains is of great significance. One possible method is to examine the results of students. Although one cannot measure the learning efficiency of the same student with the same curriculum but with two different learning methods, one can still compare the results of two groups with nearly the same parameters, who learn the same material with different methods. One group uses the examined program, the other does not, and at the end of the inquiry we measure the knowledge of the students. In this case we have to pay bigger attention to the type of knowledge we measure; we must not let the used learning method affect our expectations (the questions we ask). If this process cannot be attained, but we have enough elements, we can compare the results of the same students learning different curriculums, based on the fact whether they have used the programs in the given semester or not. We can also compare the results of different student groups with the same background and curriculum, those who used the programs and those who did not.

Instead of objective evaluation we can also use a subjective one; we can explore the opinion, satisfaction and experiences of the users, either with questionnaires or through interviews or focus groups. Comparing the operation and structure of each program can be another useful method of study. The so-called "checklist" method mixes the latter two: this is when the evaluator has a list of features to be evaluated in connection with the program in question. This is one of the most popular evaluating methods. In this case usually teachers who use the program themselves give an evaluation about it, answering the questions of an elaborated list of possible features and attributes of the program [1].

2.3 Experiences with WebAssign

Among other inquiries of efficiency of e-learning systems (see [5]) I made a survey in the American International School of Budapest in the fall of 2011; I wanted to find out how efficient do they find WebAssign, an online homework-making program. In this institute it is not the high number of students that makes the e-learning system necessary; they use the program out of the firm belief that this method is more suitable for handing over the knowledge of mathematics to the students.

I charted the functions of the program as part of the survey, and I mapped the opinion of teachers and students; the former with structured interviews, and the later with survey method.

WebAssign is a software developed for teaching mathematics; with a suitable syntax practicing at home with computers can be much more than merely solving tests. When solving a problem, the entered answer can be a number, and also a function, an interval, a formula, etc. Therefore one can use the program to solve such algebraic conversions, equations and inequalities that can cause difficulties in high school education, and to drill countless types of mathematical problems. At the same time it is not really suitable to teach mathematical theory like proofs of theorem, or to make geometric constructions. Problems which require creative efforts must stay paper-based, and their supervision must stay in the teacher's hand. Geometric constructions on the other hand probably will soon become executable and controllable electronically, thanks to the constant developing of software. In short, the program has its own limitations; it can be further improved with software development, but most probably will never be suitable for teaching complex mathematical thinking. Nonetheless, it has never been its goal.

The software's goal is to help the average student to practice certain types of problems at home. This function is relatively simple, and supported by lots of useful features:

- Each problem can be solved many times, with different parameters.

- Errors are displayed immediately, so students can retry and rethink the problem to find the correct answer.

- The teacher does not need to check all home works personally to see how many attempts it took to solve a problem, and which ones were not solved, so he can decide what needs more practicing or repeating.

- Students who are clueless or doubtful about how to deal with a problem can watch a video showing others solving the same task with different values. (This opportunity is unfortunately not available for all instances.)

The preliminary answers given by students to the attitude questions of the questionnaire are shown in the table below (N = 89):

It can be marked out from the questionnaire that students generally like to use WebAssign. The chance of immediate viewing of results (the instant feedback) is considered a serious benefit.

The most frequently mentioned doubt in interviews and conversations about e-learning programs is that the knowledge gained on computing interfaces will not be usable in other environments, for instance in a traditional paper-based one. But students questioned in this questionnaire did not mention to have the problem of being unable to solve mathematical tasks on papers they otherwise are able to solve with the computer. It is an important point to emphasize, because in certain cases of similar programs (the WeBWorK program for example) this problem has arisen. If the types of questions the program asks are limited, even if within a topic, it can happen that the strangely put questions of the paper-based tests can confuse the students. WebAssign probably avoids this issue by having a good user interface.

The method of showing mathematical terms is also important. It is a key requirement of similar programs to show problems, answers and terms in a (visual) way that is commonly used in mathematics. If a student enters xˆ2 as a solution, then x^2 should appear on screen, just as it would look like if written on a paper.

So one of the most important usability question (which also causes the most problems and tensions) is the issue of syntax or programming language; how the user communicates with the program, how he enters terms, mathematical signs, algebraic expressions, intervals, etc. Programs usually have their own syntaxes which users have to learn, because keyboards are not specialized. Half of students questioned about WebAssign considered it a problem. This technical issue needs to be handled, because if fixed, the usability of the program would strongly improve. Even if someone is familiar with the notations, it can still occur (especially in international schools) that a new computer's keyboard settings can causes some difficulties.

Variable	yes	neutral	no
I am satisfied with WebAssign	63	17	20
Using WA makes it easier to understand Mathematics	43	34	23
By using WA I learn only to solve several types of problems, while the theory remains unclear	33	34	33
Using WA makes it harder for me to understand theory	15	25	60
WA helps me more by gaining Mathematical knowledge than the “traditional” ways of learning	28	41	21
WA makes Mathematics courses more enjoyable	38	29	33
I often get frustrated with the syntactical requirements of WA	40	28	32
The immediate responses I get from WA help me learn the class material	51	39	10
The immediate responses I get from WA make me more persistent with assignments	63	26	11
I often get frustrated with the time it takes WA to respond to answers you submit to it	26	23	51
Having difficulties with solving a problem on paper which you practiced and could solve with WA	16	27	55
I prefer WA over paper-and-pencil HW	49	21	30
WA forces me to keep up with the class material	43	41	16
WA problems are challenging	50	40	10
Class lectures effectively prepare me to complete WA homeworks	57	29	14
The content of WA homeworks is consistent with the material taught at classes	64	29	7
The content of WA problems is consistent with the material tested on exams	52	37	11
WA effectively prepares me for examinations	50	27	23
I often get frustrated with and give up on a particular problem due to mathematical difficulty	18	30	52

It is rather a question of contents that the asked students does not feel that WebAssign only makes them practicing problems of common types, and lacks in giving theoretical knowledge. The key here is the intelligent usage of the program by the teachers; they must avoid reclining only upon the program, they must balance its obligate simplicity by giving more complex, elaborated problems to solve. They also have to follow an adaptive behavior during the personal meetings, the actual classes.

I consider it important to emphasize that the students thought the homework suited the curriculum; it would be a mistake to think that is evident. You need the problem libraries to be abundant, searchable and manageable, so the teacher can find the suitable problems to be solved, and you need him to be busy in learning the database management, and to create new problems when he has not found the right one. It takes time and energy; in those institutions where e-learning systems were introduced for saving resources it can be a serious disadvantage. Although theoretically this issue only occurs at the initial interval when the problem library needs to be filled, in practice educational materials are not persistent, so the databases containing the tasks need continuous management.

3 Conclusion

According to these preliminary results it can be said that overall the students are satisfied – even if not a huge majority of them – with the program, very similarly to the opinion of students of University of Calgary towards WeBWorK [5]. However, additional analysis is needed to reveal the details of these attitudes and to find out if there are special groups of students who can use the program with significantly higher or lower satisfaction/results than the others. At the same time, the application of other evaluating methods is desirable to elaborate the results, and to reveal how valid the results of such an opinion survey can be.

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Digital competence and its impact on student performance results analytical comparison of digital competence within Slovak, Czech and English mathematics

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Abstract. Actual social and commercial situation comes across as recessive. One of spheres directly influencing this condition is an educational sphere and vice versa unflattering decadence manipulates the sphere of education. Important part is to determine such relevant limits and stimuli correctly which cause quality of its system. Focus on digital competence content as one of stimuli participates and offers the ways for knowledge level increase and development of student capabilities within specific educational disciplines (Gazdíková, 2010). By analysis of explicitly-defined capabilities reflecting the focus of digital competence within a comparison of selected European countries; we reflect a cross-sectional perspective on contentual disproportion of capabilities regarding to digital competence. Reciprocally analysis presents relative contentual interference from the point of educational policies of analyzed nations.

Keywords: Average Scale Score, digital competence, Information and Communication Technologies, mathematics, national curricula, standardization

Classification: R30.

1 Brief look on student performance results

Nowadays educational sphere is concerned in varied student performance measurement programmes at national and transnational level oriented to evaluate student level of various literacies. Global comparisons intent on level of disciplinary knowledge demonstrate great differences in student performance results and interrogate about knowledge policy adjusting of particular nations and its functionality.

Group of specific subjects within different levels of study presents sphere of ultimate interest due to their potential for a prospective profit-admitting. Global drive of constant improvement of conditions positively influencing student knowledge results in generation of transnational programmes measuring student performance. Programme for International Student Assessment (PISA) as a transnational programme accentuates evaluation of subjects as Science, Mathematics, Problem-solving activities and Reading. Trends in International Mathematics and Science Study (TIMSS) similarly to another student assessment reports at various level of education exceed political and commercial intentions but academic purposes internally managed by revision of statutes within national knowledge policies as well; to concretize limits and stimuli of contemporary national educational conditions. According to TIMSS comparisons realized in 2007 at 4th grade there are excessive differences within European countries (inter alia in examined Mathematics Content).

Average Scale Score (ASS) in Mathematics Content of England denotes 2.09 percent increase in compare with score of Slovak Republic and 3.61 percent increase in compare with score of Czech Republic. (Figure 1) England proves primacy not only

Figure 1: Multiple comparisons of Average Scale Score within selected European countries (Source: TIMSS 2007)

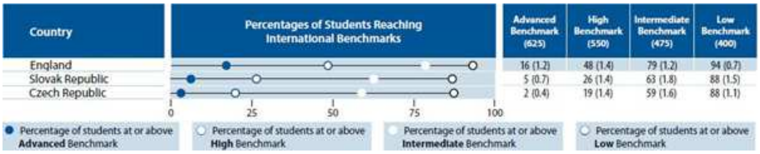

Figure 2: Percentage of Students Reaching International Benchmarks (Source: TIMSS 2007)

of the three selected countries but confirms knowledge functionality of its students in multiple comparisons within other European countries as well. ASS as statistical data cannot judge particular national educational system but may indicate progressive qualitative tendency of educational policies regarding to innovative approach using trends within sphere of Information and Communication Technologies (ICT).

Figure 2 presents four groups of students reaching international benchmarks, percentage of which differs expressly. Percentage of Students of Advanced Benchmark (AB) in Mathematics Content of England denotes 11 percent increase in compare with score of Slovak Republic and 14 percent increase in compare with score of Czech Republic. Percentage divergence of English, Slovak and Czech students reaching High Benchmark (HB) and Intermediate Benchmark (IB) is evident as well.

ASS as statistical data cannot judge particular national educational system but may indicate progressive qualitative tendency of educational policies regarding to innovative approach using actual operational trends. Advancement of ASS (Fig-

Figure 3: Advancement of Average Scale Score resulting in 1995, 2003 and 2007 (Source: TIMSS 2007)

ure 3) presents future tendencies where (without reference to concrete ASS results) improvement of English results is visible. Czech ASS allocates negative addiction (11 percent decreasing). Absence of Slovak students in 1995 and 2003 disables to present future tendencies.

2 ICT and student performance

Nowadays Information and Communication Technologies (ICT) represent an irreplaceable component in all of educational spheres and grades. (Gazdíková, 2005) As far as transnational social performance measuring programmes are concerned volume of ICT implementation is crucial. Multiple comparisons of average student achievement within a primary school mention the fact that in specific fields of subject student measure of performance differentiates at any level of education. Despite of ambition to internationalize education systems of European countries revision of statutes inspired by Lesson-drawing, Policy emulation and other multinational mechanisms does not involve potential of ICT sufficiently. (Virkus, 2003), (Holzinger, Knill, 2008) As far as potential of ICT as technical support for educational process is mentioned, digital competence as a frame of mans capabilities has to be introduced.

2.1 Digital competence in short

Mans competence to do something closely interrelates with academic standardizations to the question of its content crossed by political and social background. (Holzinger, Knill, 2008) European social legislation defines brief explanation of digital competence content which may be partially considered as academically aimed towards functional knowledge policy within nations of European Union. (European Council, 2006) Digital competence as one of key competences includes set of knowledge, abilities and attitudes focused on usage of technologies of informational society for working, spare time and communication as well. The aim is to develop cognitive sphere by acquiring of knowledge about character, assignment and using opportunities of ICT. Development of abilities by acquiring of digital competence is based on cognition and usage of ICT for manipulation with information, their editing and processing and communication, which support cooperation in cultural, social and occupational sphere as well. In compare with other studies dealing with digital competence content is practically identical and in fact accentuates three main fields of focusing- field of information, field of communication and field of technical support. Within these fields academic purposes - ethical usage, creative potential, information processing or critical thinking are complexly developed.

3 Digital competence in sphere of mathematics within Slovak, Czech and English educational systems

Comparative analysis concretizes digital competence content as content defined in national curricula for a field of Mathematical studies and presents relational contentual interference within nations analysed. To analyse quantitatively representation of digital competence and to synthetize its form we incorporate a standardized capabilities according to International Society for Technology in Education divided into six spheres. They complexly develop system of knowledge and abilities of student from the sphere of ICT - data processing, construction of new informational knowledge, communication and cooperation, critical thinking and digital citizenship.

SUBJECT	NATION	Content of digital competence (ISTE NETS standardization)						
		Creativity and innovation	Communication and cooperation	Data- processing	Critical Thinking	Digital membership	Technological operations and concepts	**Quantitative summary**
Mathematics	Slovak Republic	x	-	x	-	-	x	**3**
	Czech Republic	x	-	-	-	-	-	**1**
	England	x	x	-	-	-	x	**3**

Figure 4: Comparison of digital competence content proportion within analysed nations

(ISTE, 2007) Contentual focus of particular spheres ought to help students to develop capabilities and to construct informational literacy necessary for their occupation. Regarding ISTE standards measure of digital competence capability implementation represented in national education curricula of analysed nations presents notable disproportion of particular capabilities explicitly defined for subject of Mathematics (Figure 4).

Creativity and innovation as first sphere of digital competence is explicitly defined within national curricula of all analysed nations. (ISCED1, 2008), (VUP, 2007), (MSMT CR, 1995), (NCfE, 1999) Development of creative thinking and innovative output production by usage of ICT within Mathematics is in case of Slovak curricula reflected in development of student capability to process data adherent to use of calculators. (SVP, 2009) ICT may support searching and data-processing in form of figures, charts and diagrams. In case of Czech and English national curricula, student capabilities explicitly defined are formed and aimed to develop data-processing and interpretations of results using various implements (physical objects, models, visual presentations etc.). (VUP, 2007), (MSMT CR, 1995), (NCfE, 1999)

Usage of digital media and communicational setting for communication, individual learning support or support of other student learning as second sphere of digital competence is for subject of Mathematics explicitly defined only in English national curricula, although there is theoretical reference about its incorporation in externalization of study programme in Czech and Slovak curricula as well. (NCfE, 1999), (VUP, 2007), (ISCED1, 2008) English curricula concretize sphere of communication and cooperation defining a focusing on development of mutual communication- Student by usage of informal language and record (in spoken, visual or written form) and consequently mathematical language and symbolization communicates with others. At that he uses ICT as an implement for result-linking by using symbols too (NCfE, 1999).

Third sphere of digital competence is explicitly defined within national curricula of analysed nations disproportionately. Application of digital tools for data acquirement, evaluation and utilization is in subject of Mathematics explicitly defined only in Slovak national curricula. Theoretical definition is reflected (within aim of subject definition) in form of ICT usage for data-processing (SVP, 2009) without any fractional concretization of its realization.

Critical Thinking and Digital Membership as fourth and fifth sphere of digital competence are not explicitly defined within national curricula of any analysed nations. (NCfE, 1999), (VUP, 2007), (ISCED1, 2008)

Technological operations and concepts as sixth sphere of digital competence is explicitly defined within Slovak and English national curricula aimed to sphere of Mathematics. (ISCED1, 2008), (NCfE, 1999) Slovak curricula define its realization focusing on student selection and productive usage of ICT (software applications, etc.) applicable for solving of specific tasks and problems. ICT directly participates on student digital skill development (by usage of calculators, software applications etc.) through data searching and processing. (SVP, 2009) English national curricula theoretically reflect capabilities adherent to sphere of technological operations and concepts in productive usage of analogue or digital equipment for specific task solving (inter alia calculators for numeric task solving) as well. (NCfE, 1999) Student may understand problem of angles and radius and develop his skills by usage of software applicable to object design of alternative space solutions or solving of numeric tasks as well. (NCfE, 1999)

4 Conclusion

Information resources and capacity to understand and use ICT accurately impact both qualitative and quantitative assurance of knowledge of volume-related subjects as Mathematics due to their wide integration opportunity. Representation of digital competence within national curricula is in sphere of Mathematics defined insufficiently. Comparison presents quantitative differences of defined digital competence (according to spheres defined by ISTE) intensified by unequal definition of its content. Digital competence arrangement within selected European countries influences their educational systems qualitatively resulting in excessive ASS results. Transnational communication and cooperation with nations achieving high ASS results supporting by analysis of practical digital competence development (based on ICT implementation within specific subject) used in such educational systems may positively influence quality-raising of national educational systems of other analysed nations.

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The role of knowledge acquired in secondary school in higher education

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Abstract. We have been witnessing more and more depressing results concerning the abilities of students joining higher education lately. Their knowledge of mathematics is worse than it used to be, and it is not sufficient to support a successful professional training. The results of mathematics tests carried out among college freshmen reflect a decisive contribution of the knowledge acquired in secondary education to the success of students in higher education. We conclude that the mathematical prerequisites of admittance to a college should be clearly outlined, and that students who intend to join higher education should take the advanced level final examination in mathematics.

Keywords: maturity, preliminary knowledge, problem exploration.

Classification: D60.

1 What led to the thought of researching this topic?

The global economic, social and technological changes directly test the adaptability of humans. The endeavour to meet this heightened competition and the changing labor market needs represent a challenge for the education system. The European Union wanted to answer this in its Lisbon decision of 2000, in which it formulated the requirement for reforming the education and training system of member countries in a way which creates the basis for the knowledge-based society.[1]

The Council of Europe intends to change the education and training system so that it focuses on competences, skills, and the ability to apply knowledge instead of the traditional content-centered teaching. Accordingly, new curricula, new teaching methods were introduced, and competency-based education as a concept has become widely accepted. In contrast, our mathematics teaching, which has been outstanding even by international comparison in previous decades, has declined. The knowledge of students entering higher education, despite of their high admittance score, is significantly lower than in the past, and they do not have the solid basis on which the specialized training of a higher education institution can be successfully built.

At the Budapest Business School, in order to build the foundations of specialized subjects, the course of Economic Mathematics is indispensable. This block contains analysis, probability theory, linear algebra, and operations research. These objects require solid mathematics skills. Right from the first semester, students begin with analysis, where they need a stable and thorough knowledge of algebra, equations, inequalities, and functions.

2 The Placement Test

Like every year, at the beginning of the current academic year students wrote a placement test which complies with the requirements of the intermediate level high school leaving examination, compiled along unified principles. The test was created

[1] The conclusions of the presidency. Lisbon European Council, March 2000, 2324., point 26.

to assess the knowledge level of the incoming students, and to form a targeted catching-up strategy.

On the first seminar more than 1000 students wrote the test, of which 808 have been processed.[2]

89% of the students have an intermediate level maturity exam, and two students admitted that they did not take the exam in Mathematics, that is, they did not graduate in Mathematics. This fact reflects that Budapest Business School is not chosen by math-loving or knowing students, but the ones who want a degree in business.

	Distribution based on maturity exam level			
Type of maturity exam	intermediate level	advanced level	old maturity exam	no maturity exam
Number of students	722	79	5	2

Table 1: The distribution of test results based on the level of high school leaving examination results

The test focuses specifically on high school mathematics core material[3], classic mathematical skills, as well as on investigating whether the knowledge required in higher education and in order to get ahead is at the disposal of students or not. The tasks have a purely mathematical nature, and are not related to mathematical problems arising in life.

Raising to a higher power, operations with algebraic fractions, equations, inequalities, defining the definition sets and value sets of functions, were all tested. There were 10 different exercise sets of the same kind, a unified key, unified scores, and 25 minutes solving time.

Placement Test 1:

Task 1. (2 points) Simplify the following fraction ($a \neq 0$)

$$\frac{\left(\frac{1}{a}\right)^{-\frac{1}{2}} \cdot \sqrt[3]{a}}{\left(3a^{-2}\right)^2 \cdot a^0} =$$

Task 2. (2 points) Carry out the operations by considering the possible values of the variable!

$$\left(\frac{4b}{2b+5} - 2\right) : \frac{5}{4b^2 - 25} =$$

Task 3. (2 points) Express the value of t from the formula $4^k = 5^t$!

Task 4. (3 points) Solve the following inequality for real numbers!

$$\frac{\lg(2x+10)}{2} < \lg(x+1)$$

[2]The chosen pattern is not entirely representative for the students of all economic colleges.
[3]Decree number 40/2002. (V. 24) about the detailed requirements of the maturity examination.

Task 5. (3 points) Enter the most extensive subset of real numbers, where a function can be interpreted with the following rule:

$$f(x) = \sqrt{2-x} + \frac{1}{x^2}$$

Task 6. (4 points) Function f is given, interpreted in the set of real numbers:

$$f(x) = 7 - |x+3|$$

a) Where does the function have a value of 6?

b) Define the value set of function f!

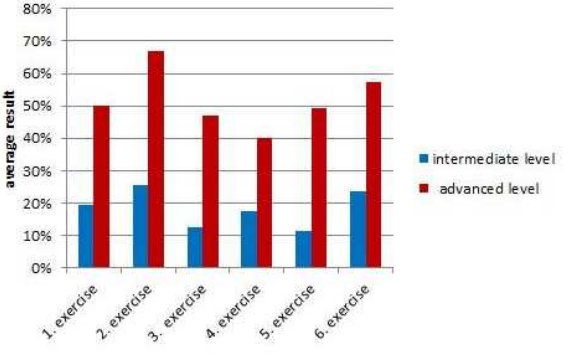

Figure 1: The results of individual tasks

The least likeable task was the third task since the fear from lettered expressions appeared here. But not knowing this kind of task entails that the student will not be able to cope with tasks and formulas of an economic nature later on. Because of the lack of the function table, which was used permanently in high school, they did not have enough knowledge to solve the first two tasks. Shortcomings in function theory, namely defining the interpretation domain and the value set caused the poor results in tasks 4 and 5.

General and very common mistakes:

- Incorrect algebraic transformations: squaring by members, square root, coming up with the incorrect product, simplifying, for example:

$$2^{-1} = -2, \quad \left(3a^{-2}\right)^2 = 3a^{-4}, \quad \left(\frac{4b}{2b+5} - 2\right) \cdot \frac{(2b-5)(2b+5)}{5} = (4b-2)\frac{2b-5}{5}.$$

- Equations, defining the domain of inequalities, for example, in case of the square root equation, when inspecting the expression below the square root, they do not say anything about the sides, and do not compare the different results obtained with the result.
- Incorrect quadratic inequality solution: $x^2 > 9 \Rightarrow x > \pm 3$ or $x > 3$.
- The role of the zero function, for example division by zero.

The above-mentioned problems, the frequent appearance of false analogies, incorrect solving of operations, result in incomplete knowledge and inadequate mathematical knowledge.

Unfortunately, there was a fair number of students who committed annoying counting mistakes, but their thinking was correct. Sadly 15% of students achieved the desired 50% minimum level.

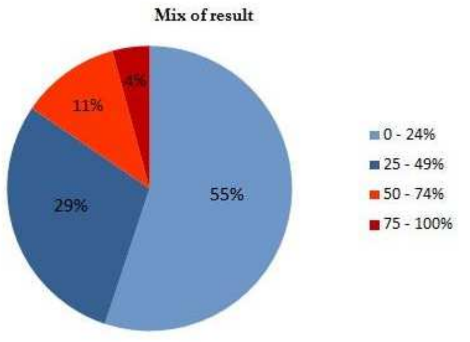

Figure 2: The results of the tests

This very poor result can be explained by the prohibition of using the function table, and by the fact that they received fewer points in the sub-tasks than they would have received in high school, or on the maturity examination.

If we examine students' high school leaving examination result levels, we find that the students who took the advanced level maturity examination performed significantly better than the intermediate[4] level ones. These students study in higher number of hours, and have a more demanding curriculum.

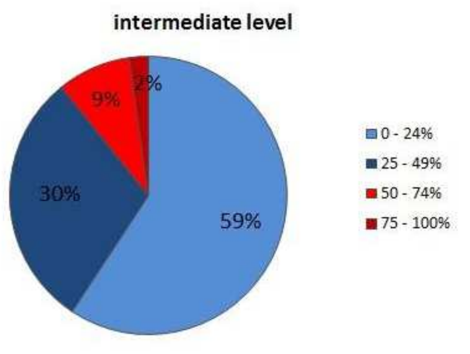

Figure 3: The results of the students with intermediate level maturity exams

[4]In case of the advanced level exam, the study group has a lower number of elements, so the resulting conclusions should be treated with caution.

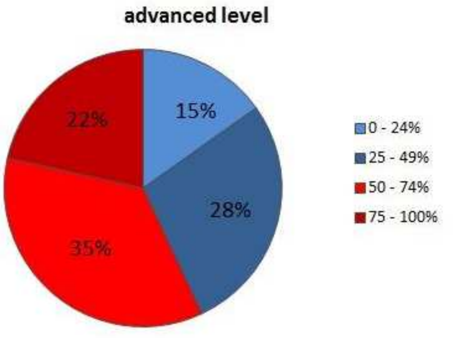

Figure 4: The results of the students with advanced level maturity exams

All the Hungarian higher education institutions are experiencing the above problems: similar results were obtained at BME (Budapest University of Technology and Economics)[5], where from 2006 students are been admitted have to write the placement test as assessment zero. Here too, only 25% of students reached the 50% level, which is the satisfactory level, and the better results were among students who took the advanced level maturity exams.

The placement tests of Eötvös University are more complex, time consuming, the material needed to proceed is assessed, and similar results were born.

Each survey clearly shows that the entrance exam score does not measure students' knowledge level, which is poor; their performance is mixed, and most importantly, the intermediate level maturity exam is not sufficient for further studies.

In every higher education institution the failure rate of first-year students is so high that it would be worth reconsidering the goals of high school mathematics education and those of the two-tier maturity exam system.

3 What striking changes have occurred in the current secondary education system in recent years?

- The introduction of competency-based education and the two-tier maturity exam In Hungary competency related projects were launched in 2004, and they were linked to the introduction of the two-tired maturity exam. The Office of Education determines the graduation requirements in mathematics at two levels:
  - intermediate level: it requires the mathematical knowledge of the people who are able to inquire and create in today's society, which means knowing and applying mathematical concepts and theorems in practical situations.
  - the advanced level contains increased-level requirements, but the requirements set out in the same way are achieved by more difficult tasks, which require more ideas to solve. In addition, among the specific requirements

[5] Anikó Csákány, BME Mathematical institute, Mathematical Survey

of the advanced level exam are special topics, meant primarily to prepare learners in higher education.

However, as a consequence of the higher education funding system the intermediate-level school-leaving examination was sufficient for the entrance exam. Currently, the higher education plan contains a new maturity system, which is being worked out by professionals but it seems that the changes will occur in the selection process and not the system requirements within the subjects[6].

- Lower system requirements for students On the maturity exam a student can receive a pass grade if:
  - A minimum of 20% is achieved by him/her on the intermediate and advanced level exams.
  - Can adequately use the function table and the computer. This is creative problem solving, but we do not need this.
- Decreased time frame for teaching If we look at the time frame for teaching, it, we find that the total number of hours in all the education systems are falling, while the volume of the syllabus increases. Balancing the amount of the material, its modern processing, and the time allowed for this seems insolvable.
- Changes within subjects
  - New topics have been introduced, for example probability theory, mathematical statistics, graphs, which would not be a problem if the principles of moderation were kept and it would not detriment mathematical operations, such as algebra transformations and geometrical thinking, which are of basic importance.
  - Because of content regulation and the disproportionate distribution of the syllabus the part of topics which is indispensable for acquiring a higher-level mathematics knowledge has been decreased. Algebra, geometry and analysis are present at a proportion of 20-20% each.

set theory, logics, combinatorics, probability theory and statistics	40%
algebra, arithmetics, number theory	20%
functions, elements of analysis	20%
geometry, coordinate geometry, trigonometry	20%

Notice that in the description of the graduation requirements in Mathematics we often find the "graphic" expression, which signals that a lot of mathematically precise knowledge is not required, and instead the "little bit of everything" principle applies.

  - The change in approach requires a greater emphasis on literacy (understanding text) and language skills. 30-40% of the tasks is textual, connected to everyday life situations, and requires model-making tasks. It should also be noted that students' native language comprehension skill is in a pathetic condition, often the children cannot arrive at mathematical problem solving because they do not understand the text.

[6]http://www.oh.gov.hu/valtozasok-felsooktatasi

- Some objective factors, such as mass education and classes with a large number of people prevent the application of knowledge acquired in high school.

4 Emerging dilemmas

Statistics show that the practice-centered approach to the mathematics maturity examination has not made things better. The average points of students decreased compared to results prior to 2005, and in higher education more and more depressing results have been achieved. To begin with, you should ask the question of whether the competency-based mathematics tasks are sufficient enough to graduate from a higher education institution while the students do not dispose of the scheme system of the most significant mathematical concepts and operations.

- Will students have a more applicable knowledge by solving problems related to economics, IT or even everyday tasks, while they are unable to perform basic operations?
- Will students be able to solve the mathematical problem if we consider the lack of comprehension - literacy, understanding text - skills?
  E.g. elevated level maturity exam /May 2007 / Task 6:
  The weight of the plum seed is about 5% of the mass of the ripe plum. When stewing plum, during the drying technology water is subtracted from the unseeded plum until only 5% of the remaining weight is made up of water, what remains is the dry matter content. The resulting product is called prunes.
  a) Considering the above, show that from 10 kg of picked plums 1 kg prunes can be produced!
  b) Mr. Kovacs has sold half of his plum crop as raw, the other half as prunes. How many kilos of plum was his crop if the raw and dried plums amounted to a total revenue of HUF 286000 altogether[7]?
- Do the new four-digit function tables help students learn the routine operations?
  The motto of competency-based education is: knowing basic concepts, theorems, as well as recognizing and applying them in practical situations. Using the new function tables in every situation completely contradicts this theory because if students do not learn the theory, definitions, rules, theorems, then their theoretical background will not be enough for the solution patterns of the tasks.
- Is the content modernization equivalent with the tasks given from the new topics in a disproportionate manner?
  For example, in 2009, 42.61% of the advanced level maturity exam consisted of set theory, combinatorics, probability calculation and statistics. (tasks 2, 6, 7 / c., and 9)[8].
  This proportion is valid for the maturity exam every year. The new topics being introduced represent a high percentage of the tasks, and it seems that knowing these precisely is not enough to graduate a college or a university.

[7] The whole task set and the correction and assessment notes can be found at http://www.oh.gov.hu/letolt/okev/doc/erettsegi_2007/e_mat_07maj_fl.pdf

[8] http://www.oh.gov.hu/letolt/okev/doc/erettsegi_2009/e_mat_09maj_fl.pdf

The first obvious fact is that we should strive for a uniform distribution of different subjects. Knowing certain bigger topics is not enough to achieve the sufficient level. So the content regulation has brought to the surface new problems, which are more serious than any former ones, and I consider important that further research is carried out in this direction.

5 Conclusion

Change does not necessarily mean improvement; introduction of new topics does not automatically lead to better understanding.

The intermediate level examination requirements are adjusted to the general level of education, while in order to enter the higher education system, the advanced level maturity examination is needed, this is confirmed by results of the placement papers, since those who learn mathematics by their choice perform much better. So high school students preparing for mathematical studies in higher education need skills and stable specialized knowledge to be able to solve problems within the subject, so the effectiveness of higher education is primarily assured by requesting the advanced level maturity exam. The advanced level task set besides measuring certain competences also measures concrete material knowledge, and resembles the traditional, old maturity exam more. If the advanced level maturity exam replaced the entrance examination, like the creators of the system thought, there would still be need for change.

I agree with the intent to introduce more practical mathematics in order to increase students' interest, motivation to improve, but this should be achieved within a framework which does not reduce first-year students' survival chances. On the one hand those areas that improve creativity, model making and thinking, should keep their accentuated role, on the other hand, there is need to acquire skills in mathematical operations, equation solving, function illustrating, characterization, and skills must be acquired in plane and space geometry.

Secondary schools do not have to teach the material of higher institutions in advance, but they need to provide a foundation which effectively supports the easier acquisition of future knowledge.

If the advanced level maturity exam is not required, for this serious problem a possible therapy would be to help students catch up with the missing knowledge. Today, most universities offer leveling courses in which basic material that is needed in college is revised. At BGF-PSZK during the first semester I held two hours weekly in order to make up for the high school material. The need for catching up is great from students' part, and because of this many people attend these seminars. The effectiveness of the courses has not been inspected yet, but it is my intention to carry out such research. Since this course takes place parallel with teaching the first part of Economic Mathematics, I consider important the development of a curriculum by which mathematical deficiencies can be remedied, avoiding backlogs in new material. In the coming period, within the framework of a survey I would like to develop a strategy which helps the transition from one education system to the next, for students and teachers alike.

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Abstract.

"Learn everything you learn for yourself... " Petronius

In mathematics education there are a lot of important questions to response. I am searching answers for two questions: How can we make practice our students? How can we control their knowledge? In an ambient where we can use IT technologies in the teaching I am looking for programs that make practice students and/or can control them. I am researching among free (or almost free) programs to access everybody. To understand better programs I am writing a program, too: Kvizy. In this article I introduce/compare some computer survey programs: Kvizy, iTester, Hotpotatoes and some teaching programs: Hotpotatoes, www.maths.hu, KhanAcademy.

Keywords: Collections of computer programs, Control of knowledge, Examination, Computer assisted instruction, Technological tools, Survey, Secondary school, University, Adult education, Higher education.

Classification: N80, D60, U50, U70; B68, B40.

1 Introduction

I am a PhD student so I am studying and teaching at the University of Debrecen at the Department of Applied Mathematics and Probability Theory.
I know the time we posse to teach students is very short and the material is getting bigger day by day. Looking at these problems I decided to search programs on internet (to self-study and/or to control) by two criteria:

- it can be used by students of secondary schools, universities or adults,
- it will be free or low-cost to be able to access students or everybody.

The first is because pupils more than 15 years have maturity to understand they are studying for themselves and they can use computer without problems.

The second is because, by the experience, there are very good and wonderful projects and they function very well until the project has money.When the money runs out a lot of times the project dies.(For example at my university also was a very good project named *mobiDIÁK* with excellent teachers and materials but nowadays are few people using this very useful material-system. This is very sad.)
I also looked for free programs because in Hungary schools and universities are not well-paid and I am seeing amoung our possibilities.

I will mention Mathematics Didactics *(Did.)* and IT *(IT.)* aspects, too.

2 Control and teaching

1. First I want to present some programs to control. I only see questionnary-making programs or part of them.
2. In the second part I introduce some programs/homepages that have methods of teaching.

3 Control by questionnary-making programs

The control, the evaluation is almost the 'end' of teaching. One type of control is the test. By *Binet*

> "This method, that measure the level of the knowledge of the student, has three advantages: - demostrates clearly the skills of every students ... allows to measure the process of students ... with this we can measure, evaluate pedagogic methods before to use them in the live education "[1]

There are several types of tests but primarily I look for only the 'Single choice test' where there is a question and 2 or more answers. The responsive has to choose one right among many bad.

(Did.) By didactics this type of test the simpliest: one ask and one good answer. It can be problem that the responsive has luck and without any knowledge can produce a full good test. Yes, it can happen. The solution: to get more difficult test giving closer bad answers to the good and/or extend the number of answers. In the last case the probability to mark a good answer by chance will decrement with the rising of the number of answers. It will be harder to remember to a swottinger too.

(IT.) At the background of every test there is a database to store questions, possible answers and data.[2] Nowadays one of the most popular combination of survey softwares:*(PHP + MySQL)*. This combination make fast and dynamic programs.

- *MySQL* is a database-system to store and manipulate data and[3]
- *PHP* is an independent programming language can make dynamic pages [4]

'Single choice test' is, by programming, the simpliest test because we do not have to pay attention how to insert text, upper/lowercase problems also are eliminated, etc. and the main process is only compare the good answer with the given answer.

3.1 Criteria of a good questionnary-making program

(Did.) If you use a survey program as a teacher you have some main criteria:

- easy to use
- easy to add new question
- you can give the number of questions
- you can give the number of answers
- after filling the form the program check it
- it shows/compares you the good answers
- it give you statistics of result
- you can choose the subject/s

3.2 Kvizy

This program is under construction. It can use on my localhost (only) but finishing I will be upload it to a shared server.

How does it work?

You can enter with a username and password or register.Then you can choose among enable functions (e.g. have a list of questions, insert new subject, etc.).Next you can choose subject and write how many questions you want (the program automatically shows how many asks are in the selected topic). Then the program generate a new questionnary with random order of questions and random order of answers. At the end when you press *Submit* the program write your answers, good answers in red and shows statistics. (Figure 1) This program serve me to understand how can prepare a program *(IT.)* and what are the didactics *(Did.)*, technical mistakes and problems.

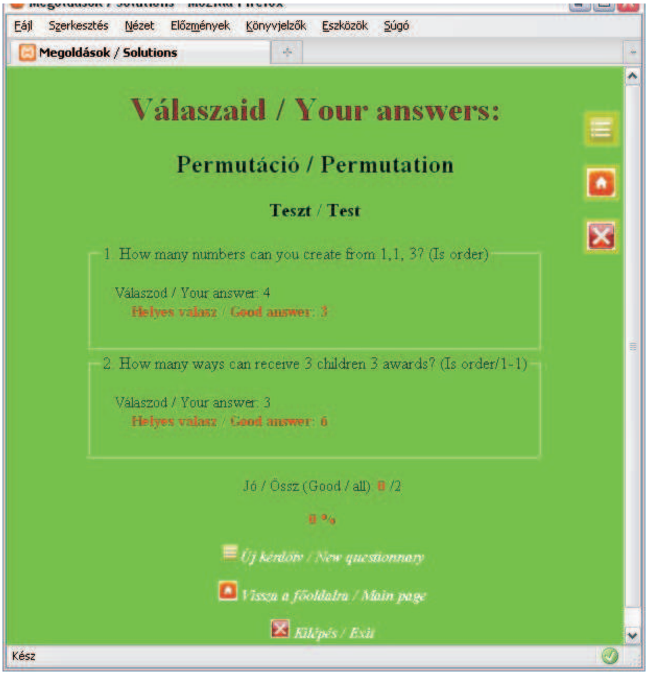

Figure 1: Kvizy - The result of questionnary

In the future I want to put a self-studying part adding solution method step by step as we can see later in the subsection of *www.maths.hu at page* 94.

3.3 iTest

This is a complete program made by *Softonic*. It is made to teachers to create tests and exams.

> "Create interactive exams and tests. iTest is the kind of program teachers love and students hate: a complete tool to create exams and tests of all kinds." [6]

How does it work?

The program has two main parts: *iTestServer* and *iTestClient*.

The Server *"which is the question/answer database editor and exam server"* [6]. This can use the teacher: you enter and give a name for your database or choose among existing databases.

You can insert new questions pressing the *Add* button.You can put a short name for the question and write a very long question at the *Question* box.You can choose among 3 marks *(Easy/Medium/Difficult)* to put difficulty, too. At the bottom you can select the type of questionnary: *Single Choose/Multiple Choose* and insert answers signing correct answer/s.

You can put similar questions with *Duplicate* button. At the end you have to *Apply* and *Save*.So you have a database. Next step is by the *Server* button where can make the really question-list clicking the *Advanced* box you can select the questions, add the *Test name*, *Number of questions*, *Time for whole test or one question*, etc. Pressing the *Start server* starts the process.

In server mode you can put the score of evaluation (percentage to pass), too. Then if you use the program in Local network clients/students can use it, if you use only on your own PC you can press *Server* button and from the menu select *Run iTestClient*.

First you can see questions one by one in full screen mode and at the bottom on right hand you can go next / last. Clicking on left column you can select the question to change the answer. When you press the *Finish* button the program alert you and if you finish you can see your answers and the result. You can print to file also. Then you can make a new test or *Quit*.

Students' work teacher can follow by server and have statistics, too. To finish the test you have to choose *Server* button and from the menu select *Stop Server*.

3.4 HotPotatoes

[7] This is a website made by Half-Baked Software, the University of Victoria and Creative Technology. This is a transition between survey programs and teaching-help programs. It has parts to study. This is a *Virtual Learning Environment* where you can use various tools.

(IT.) It is made with *(PHP + MySQL)* and they are programming now, too.
(Did.) One of the advantages of HotPotatoes: applications can use in some languages, in Hungarian, too. I think this is very important for students and teachers who do not speak in English.

> "Teachers use the Hot Potatoes programs to create educational materials, especially exercises and tests. All these materials can be produced in the form of web pages, and the web pages can be uploaded to hotpotatoes.net very simply, from within the Hot Potatoes programs." [7]

It can use without internet seeing the result in a web-browser. I mentioned "*programs*" because HotPotatoes has varios parts:*JClose, JMatch, JQuiz, JCross, JMix*. I present *JQuiz*.

How does it work?

You log in (username/password). To posee a username you have to use an application of HotPotatoes. You can download it from the official homepage *http://hotpot.uvic.ca/#downloads* selecting the Operating System. After install it you can use all applications. To make a test, insert queries you need *JQuiz*. You have to write the title of test, choose the number of question, the question, choose which type of test do you want *Single Choose/Multiple Choose/Mix/Short answers* and the answers, marking good answer/s. (It is possible insert more than 26 answers.) You can mix (randomize) the list, add picture, table in HTML form, links and text, too. Then you press *F6* to export a HotPotatoes homepage. The program offer to see the test in your web-browser or upload it to HotPotatoes.net In your browser you can fill the test, go to the next/previev question. If you start answering it will show the bad answer and do not allow you to change question until you solve the ask.

3.5 Comparison by criteria

Criteria	Kvizy	iTest	HotPotatoes
Free	X	X	X
Easy to use	X	X	X
Has an own database or easy to add	X	X	X
Number of questions	X	X	X
Number of answers (from 2)			X
Choose the topic	X	X	X
Program checks and shows result	X	X	X
Shows good answers	X	X	X
Gives statistics	X	X	X
Languages	Eng/Hun	Multi	Multi
Internet only	no	no	
Images		X	X
Single choice	X	X	X
Multiple choice		X	X

To sum up this type of programs are useful not only in maths topics but other topics too e.g. teaching languages, make IQ-tests or make lists about a given topic.

4 Teaching softwares

(Did.) [5] Teaching is a very complex process. We as teachers have good/bad habits, methods, personality, physical and psychical states/skills, have intuition, know our students, etc. A simple program cannot have these aspects only can be a useful tool in the comprehension process. *We* have to think about how to teach and with what. These programs cannot solve teaching problems only we can look at them as useful tools.

4.1 Criteria of a good self-study or teaching softwares

(Did.) To use a teaching program you have some criteria:

- easy to use,
- has clear declarations, questions, answers and descriptions,
- has subjects/topics (to group materials),
- you can follow the resolution steps easily,
- resolution steps are detailed,
- has image/s or video/s or something to help,
- can you ask, too,
- if you want to insert new material you can do it,
- interactive (if it is possible).

If a program has more possibilities is better. These are only some criteria to examine a program.

(IT.) If we use a similar program for maths problems one of the most difficult problems is how to write/insert maths formulas. Nowadays are projects to resolve this problem and to have a common standardized solution.

4.2 HotPotatoes

(Did.) [7] In one of the previous sections *(See . Hotpotetoes. on page 92)* I wrote in detail about HotPotatoes. When you do test with the program it checks the answers immediately and do not allow you to follow until you do not have (click) the right answer. So you will have the good answer at the end. Also is a good that the program marks next to the number of answer with a pictogram that is correct or not and at the same time in the middle of the page at the top shows Correct or No with statistics. In this case the student will have the right answer but s/he will not know the way to resolve the problem and in the future will not recognize or resolve a similar task.

4.3 www.maths.hu

[8] This is Gábor Hadnagy's homepage (a Hungarian maths teacher). His page is a good example *"How can we use internet in maths teaching"*. The site is in Hungarian. The goal of the page is help students of high education with practical examples. The page only has the most important definitions and theorems need to solve. Exercises are from three main topics: *Analysis, Probability and Operation Research*. There are some free samples. If you want to see more you can register and buy some credits (cc. 3 euros for 100 credits, cc. 4 credits per exercise) With credits you an see exercises and their keys step by step for 24 hours.
I give an example here from the Operation Research topic about minimum spanning tree. In the example there are 10 steps until the solution and every step has its own image and explication to show the way of resolution. (Figure 2)

Figure 2: Math - Last step/solution

Recently there are 485 exercises with keys. Gábor has an online *Digi board* so with this you can receive interactive lessons, too. The cost of use it 10 cents per minute.

4.4 KhanAcademy

[9] A page originally made to show exercises and practice a cousin but in a few time has became a system. They have a mission so they say:

> *"... we're on a mission to help you learn what you want, when you want, at your own pace."* [9]

They have more than 3000 videos and a mass of articles. It is in English but there are subtitled videos and presentations. At the main page there are always new videos and interesting conferences to see them. There are a lot of topics (not only maths) in three main topics (e.g. Math, Science, Humanities & Other). There are various levels of materials. The system uses American topics. On the videos you can see a black-board and a pen writing with different colors during the explications you can hear. The quality of script is not the best but you can follow it. You can make tests, too but exists few tests and its level is very basic for us. If you are making a test on the right can see the result *Check Answer* or *Show Solution* . With a smiley-figure they make more emphatic the result. At all page of practice the site advices you videos to practice the given topic. If you want to see a video you can do it only clicking on it. If you have a question or a comment you have to login and ask from the authors or teachers. And they will response you. With a username of Google or Facebook you can login. The program make statistics and graphs, too and you can use those. If you open the site it will collect all of your movements on page and you can see all your operations and statistics. To help their mission you can be a volunteer or uploader new videos.

> "Students: track your video and exercise progress in your profile.
> Coaches: Track all of your students with the class report."

This program is not an interactive page but almost is with a few time of waiting for the answer.

(Did.) You can ask me about bad intention, level of the material or bad materials to upload - this is a difficult question. It is depends on uploader's intention.

4.5 Comparison by searched criteria

Criteria	HotPotatoes	www.maths.hu	KhanAcademy
Free	X		X
Easy to use	X	X	X
Has clear descriptions	X	X	X
Has subjects/topics	X	X	X
You can follow the resolution easily		X	X
Resolution steps are detailed		X	X
Choose the subject	X	X	X
Has images/videos to help		X	X
Can you ask		X	X
Insert new material	X		X
Language	Multi	Hun	Eng
Interactive if it is possible		yes	yes

5 Conclusion

Summarizing these results: these sites/programs are very useful and in case of us (Hungary) - where there is good student-resource (they are very clever in general) and not so much background resources to teach - we can and we **have to** use opportunities of biggest countries. Maths is not always preferred subject for student because they do not understand and we do not have enough time to explain all to them. So they will not feel the success that mean to resolve a problem independently (as mentions Gy. Pólya) [5]. I hope that with practice pupils will have success and with new technologies new appetite to get knowledge from and for their ambient. So I am following my research with the goal: prepare a site/portal with a lot of links of useful and free survey programs/sites and teaching programs to help our teachers and students.

My work is not finished with this comparison because there are new and newer programs and with e-learning the process is infinite.

Acknowledgements

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Restriction on construction tools used

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Abstract. Dynamic Geometry Systems make it possible to create a multitude of drawings quickly, accurately and with flexibly changing the input data, and thus make the discovery of geometry an easier process. A general characteristic feature of these systems is that they store the steps of the construction, and can also execute those steps after a change is made to the input data. The objective of this paper is to demonstrate the application possibilities of Dynamic Geometry Systems in primary and secondary schools.

Keywords: teaching geometry, Dynamic Geometry Systems.

Classification: D30; U50.

1 Introduction

The emergence of computer programs in education forces people to rethink their views, as we have the reconsider what we regard as important. Formal knowledge is devaluated (since the computer can perform such tasks faster and better), and a higher value is associated with the capability for problem solving.

Computer-based drawing programs provide assistance with regard to this objective, opening up new opportunities in the teaching of geometry. They make it possible to create a multitude of drawings quickly, accurately and with flexibly changing the input data, and thus make the discovery of geometry an easier process. Dynamic Geometry Systems (DGS) are good match to problem solution process. As students draw objects and investigate relationships between objects, the representation of objects impacts the nature of the relationships between objects. DGS are good match to problem solution process.

One of approach to describe the roles of new technologies in mathematics education is to examine how they are affecting each content area. We carried out research in computer-based teaching and learning of graph-oriented problems. The objective of this paper is to demonstrate the application possibilities of DGS in teaching geometry.

2 Application possibilities

The development of students creativity is one of the most teacher dependent areas of mathematical education. Students who can face unexpected situations and solve new problems should be and will be in great demand. In this area, DGS may be useful in improving students skills. The way to problem solving in geometry starts with drawings, since we need accurate drawings to arrive at the right conclusions. There are so many application possibilities of DGS in the teaching of geometry:

- Searching for sets of points (simple search for sets; the method of omitting conditions)
- Discussion, examination of limit values
- Theorem checking
- Restriction on construction tools used

- Comparative geometry

Some application possibilities have been illustrated by few works. [3, 4, 7, 8]

The development of students creativity is one of the most teacher dependent areas of mathematical education. In this area, DGS may be useful in improving students skills. [5, 10]

3 Restriction on construction tools used

One good method of developing creativity is to approach known problems according to new kinds of rules. [6] A good example for this in geometry is changing the rules of constructions, for example by putting restrictions on the construction tools that may be used. [1, 9, 11] Let us consider three possibilities:

- using only rulers during the construction;
- executing old, well-known constructions using only compasses (Mascheroni-construction);
- a circle and its centre are given, and only rulers may be used (Steiner-construction).

Constructions to be made using only rulers are such that require points to be connected and intersection points of lines to be located. Using the latter two methods of constructions, all problems solvable by Euclidean construction can be solved, except, of course, that we cannot construct a line using the first method, or a circle using the second (Mohr-Mascheroni theorem; Poncelet-Steiner theorem).

Symbolic manipulation utility programs can support rapid exploration of patterns in algebraic reasoning; leading to discovery of important general principles. So DGS tools facilitate exploratory learning. DGS helps students make and test conjectures about properties of basic geometric figures.

When defining interactive problems, the teacher may control what construction tools are available in Cinderella, and thus creativity may be developed even from as early as eighth grade. Our objective is to execute the construction using the program.

Example 1. The Napoleon problem: construct a square inside a circle, using only compasses [8] (Figure 1). Vertices of square: P, Q, R, S.

Example 2. Given AB segment with its midpoint F and with a P point outside the segment. Construct the parallel to the given segment through the given P point, using rulers only. [1] (Figure 2)

At the same time this picture shows how we can construct the midpoint of segment AB with rulers if there is given a parallel line with segment AB.

Example 3. Double the AB segment with using compasses only. (Figure 3)

Example 4. Find the centre of the circle with using compasses only. (Figure 4)

4 Conclusion

DGS permitting a more effective development of experimental approach to mathematics, allowing for the exploration of problems and theorems is more interesting than those usually encountered, and not strictly within the syllabus. There is not much point in trying to find way of satisfying old objectives by new methods; instead, education should also strive to adjust to the requirements of the quickly changing world. The best way to decide in case of theoretical debates is an experimental

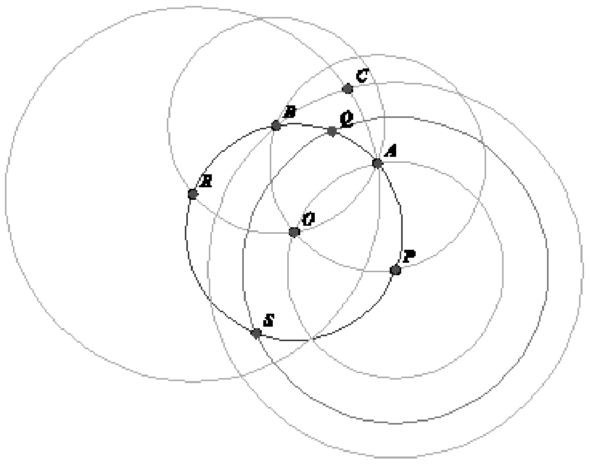

Figure 1: The Napoleon-problem

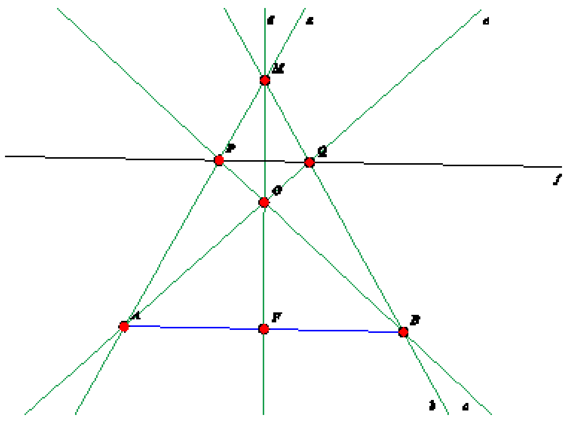

Figure 2: Using only rulers during the construction

test. It is not only the organisation of computer-aided education, but also the products thus created require special expertise; however, evaluation in a computer-based learning environment is more life-like: it is not only the extent and accuracy of students knowledge that we find out about, but also whether this knowledge will be usable.

In the computer used group it is more typical that the students helped each other, corrected their mistakes. Experimentation was more typical for them as well, as the faulty elements could be hidden without any sign with a mouse click. So the computer inspired the students to stand on their own feet. Similarly to Heugls researches [2] we also found that the testing phase at the paper-and-pencil group is often missing. Computers provide a more user-friendly atmosphere for teaching and they are more suited to the needs of the individual learner.

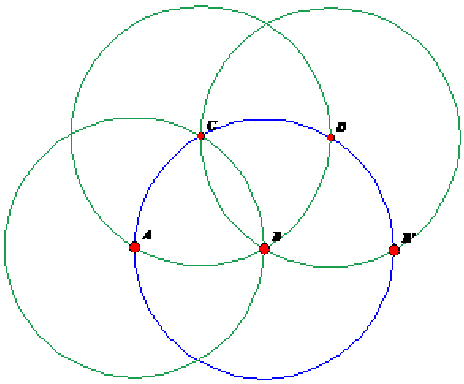

Figure 3: Double the segment AB with using compasses only

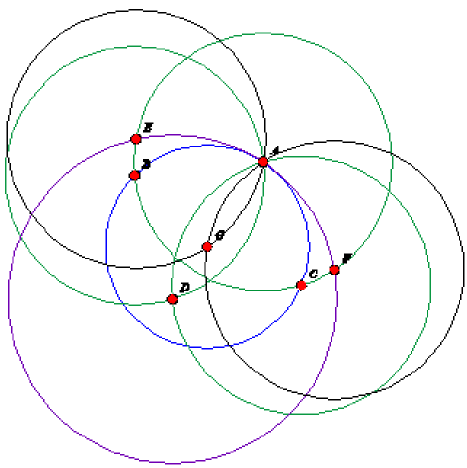

Figure 4: The centre of the circle with using compasses only

The use of computers permitting a more effective development of an experimental approach to mathematics, allowing for the exploration of problems is more interesting than those usually encountered, and not strictly within the school syllabus.

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Using educational multimedia-based software in teaching color theory to students of civil engineering

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Abstract. Electronic devices and computer software have created new opportunities in acquiring and imparting information. My goal is to introduce how to make use of our educational software in the process of teaching and learning color theory, as well as cultivating students' skills in recognizing and discerning colors. I offer a quick overview on how color theory is taught at different levels of education, and I demonstrate the results of a survey carried out at the University of Debrecen aiming to assess the level of knowledge in color theory possessed by students starting their first year at the Faculty of Civil Engineering.

Keywords: color, color theory, educational software.

Classification: U50; P80.

1 Introduction

"*Colour is life; for a world without colours appears to us as dead.*"
Johannes Itten

All human beings take pleasure in colors: it is nearly impossible to imagine our life without them. Colors surround and affect us. We can express our mood and feelings through colors. One of the goals of education (any level) is to prepare young people for a world dominated by colors: colorful pictures, posters, printed documents, movies, video games etc. Colors affect us, and through colors, we can affect others. In many cases, we use the effects of colors in modifying our environment without being aware of said effects. However, awareness of how colors can change our perception of space and mass is crucial to students of civil engineering, who need to be taught to apply the laws of color theory in practice. Below we demonstrate the application of an interactive software based on a modern educational approach, providing an active role to the student through various methods of learning. [1]

2 How color theory is taught in Hungary at elementary, high school and college level

According to Hungary's national curriculum, color theory is taught as part of the subject "Drawing and visual culture" during the first 12 years of education.

1^{st} to 6^{th} year: 1,5-2 hours per week

7^{th} and 8^{th} year: 1 hour per week

9^{th} and 10^{th} year: 1 hour per week obligatory

During the 11^{th} and 12^{th} year, students planning to take a final exam in the (facultative) subject can study it in 2 hours per week. Also, there is a period called "Arts", which could include drawing, music, drama, animation or media culture, depending on the school's decision.

In the subject ”Drawing and visual culture”, colors are dealt with in approximately 25-30 percent of the total time, which translates to 9 to 17 hours at different grades. The only way to pass on a satisfying level of knowledge on the field in the short period of time provided by the national curriculum is to apply techniques that actively involve the student in the process of learning. While learning arts at elementary and high school, students deal with characteristics of each paint type (color, brightness, temperature, saturation), and learn the basics of subtractive color mixing. They create harmonies, disharmonies and contrast. [3]

In physics classes, students learn that the white color is characteristic for compound light, which contains multiple different components (each with a different frequency and wavelength). By guiding white light through a prism, its components, the spectral colors can be separated. The total range of spectral colors is continuous: it is called the spectrum. Another goal of high school physics is to teach the laws of how light is reflected.

When learning chemistry, students observe characteristic changes of color as the result of certain reactions (reflecting changes in the energy level of chemical bonds formed by electrons).

In biology, students are taught to distinguish between visual stimuli and visual perception: they learn the basics of the wonderful physiological coordination that exists between our eyes and our brain.

At college level, students can deepen their knowledge on certain aspects of color theory that are related to their chosen field. Ophthalmologists, civil engineers, teachers and artists all study different, specific aspects that are required for their work.

3 Basic knowledge of color theory possessed by 1^{st} year students of civil engineering

In September 2011, a survey was carried out at the Faculty of Civil Engineering, University of Debrecen, to assess the level of understanding of color theory possessed by students starting their studies in the 2011-12 academic year. 125 students took part in the survey. 52 percent were admitted from a general, 38 percent from a technical high school. [7]

68 percent of the students only studied about colors as part of the arts subject. 25 percent also encountered the topic in art history, and 1 percent in hand-drawing, history of architecture, basics of printing, technical sketching, physics, chemistry, and biology, respectively.

Only 1 percent could give a satisfying definition of the color wheel. 0.8 percent knew that it contains primary colors. 43 percent managed to correctly list the primary, and 17 percent the secondary colors.

The complementary color for green, blue and yellow was correctly determined by 12, 10 and 7 percent, respectively. 12 percent of the students knew that complementary colors are found on opposite sides in the 6- or 12-color wheel.

Students didn’t show much experience in mixing colors, either. They seem to be missing any kind of knowledge on the topics of light colors and visual perception in the brain. These deficiencies need to be corrected by all means.

4 Improving students' understanding of color theory

Students of civil engineering need to understand that an important part of their future work (designing buildings) is to visualize the buildings using colors. Even in college, the number of hours spent teaching color theory is limited, so it is crucial to use the time as effectively as possible. Interactive multimedia-based computer programs are gaining ground not only in elementary and high school, but college and distance education as well. [2]

The software I introduce below was developed by us, and currently being used in teaching various subjects (Color dynamics, Sketching in Architecture VI., Expression techniques).

5 Interactive multimedia-based software to teach color theory

The educational software is useful in studying characteristic features of colors (tone, saturation, brightness), as well as summarizing experience and observing similarities between different color systems. Using the program constantly improves students' sensitivity for colors, their ability to create harmonies, their observation of how colors manifest in the inner space and exterior of buildings. It cultivates the ability to comprehend the role of color relations in expressing mass and space. Possessing and utilizing these skills is vital to modern architects, who are responsible for making our environment more beautiful to look at.

5.1 The software's development environment

To develop the program, we used the Flash ActionScript environment, which combines the efficiency of high level programming languages with spectacular animations.[6] The developed program can perform free conversions between different color spaces. It helps to understand the relations between light and dye colors, and allows changing the exterior or interior colors of a building in real-time. Test questions are also processed by a flash module, and the results are transmitted to a public server where the data are stored in an SQL database.

5.2 The program's structure

Once the program has loaded, the user can first read general information on how to use the software, hardware requirements to run the animations, and the most important chapters found on the CD. [5]

Figure 1: Initial screen of the educational software

5.3 Using interactive animations

As an example, I demonstrate a test aiming to improve the user's ability to recognize and discern colors, while allowing for a deeper understanding of the acquired knowledge through an active involvement in problem-solving.

5.3.1 Mixing colors (HSB)

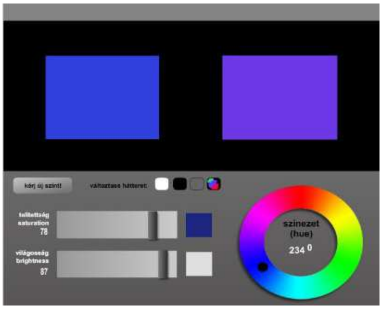

Figure 2: A color mixing experiment

The computer generates a sample color on the left side of the screen. The student's task is to match the color on the right side to the given sample, using colors taken from the color wheel below, and adjusting saturation and brightness by the sliders. The currently black background behind the rectangles can be changed to white, gray or colorful.

For beginners, the task is not an easy one: for the unconditioned eye, it can be very difficult to detect slight changes in the shade of a color.

5.3.2 Mixing colors in different color systems (HSB, RGB, CMY)

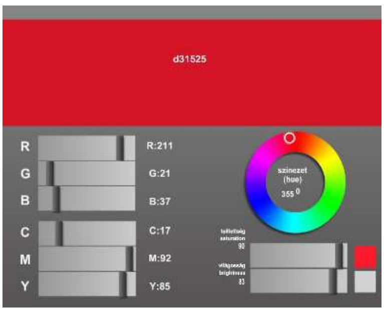

Figure 3: Mixing colors in different color systems

The educational software can simulate additive and subtractive color mixing. The student can alter the "intensity" of each color on the slides, and observe the resulting colors along with their written color code. Instructors may demonstrate in an interactive way how to mix certain colors by changing the parameters of tone, saturation and brightness.

5.3.3 Matching color stimuli

There are many possible experiments to test if the user can correctly match color stimuli, and in each one, either the brightness or the saturation differs. The program generates samples randomly, and the difference value between the colors to be compared is also random, but upper limits are constantly reduced to increase difficulty.

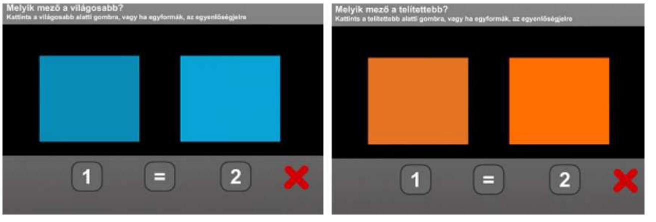

Figure 4: An experiment to match color stimuli

5.3.4 Color harmonies

When choosing colors for anything, the most important rule to keep in mind is that there are no bad colors, only nonmatching ones. Well-chosen colors can create harmony and express power. Creating a harmony of colors is essential in the design of documents and buildings alike. Students are expected to learn the way different color harmonies (monochrome, dichrome, trichrome, tetrachrome, analogous) are created and applied in practice. [4]

Figure 5: Using monochrome harmony on an interior or exterior surface of a building

Using a color chosen from the color wheel, and input values (saturation, brightness) from the sliders, the software generates a 5-hue monochrome pattern. Color codes are written for each hue. The resulting pattern can be observed immediately on an interior or exterior surface of a building. The sample hues include a darker, a brighter, a more and a less saturated version of the chosen main color.

In the example below, the type of harmony (dichrome) and the chosen complementary color pair (green and red) are the same, but there is a considerable difference in saturation values (100 percent and 26 percent). The program offers the opportunity to perceive the different effect of applying clean, saturated colors or

Figure 6: Using harmony on an interior or exterior surface of a building

lighter, unsaturated colors on the same external or internal surface. Students can experiment to find the right brightness and saturation values for coloring the interior or exterior surface.

When creating a tetrachrome harmony, the user can choose from 20 different hues altogether (5 hues for each chosen color). The software can spare one a fair amount of time, since the sketch of the building doesn't need to be redrawn each time a new color pattern is applied. There are unlimited possibilities to modify coloration, and each modification can be undone right away, so the user can test and observe many different patterns in a matter of seconds.

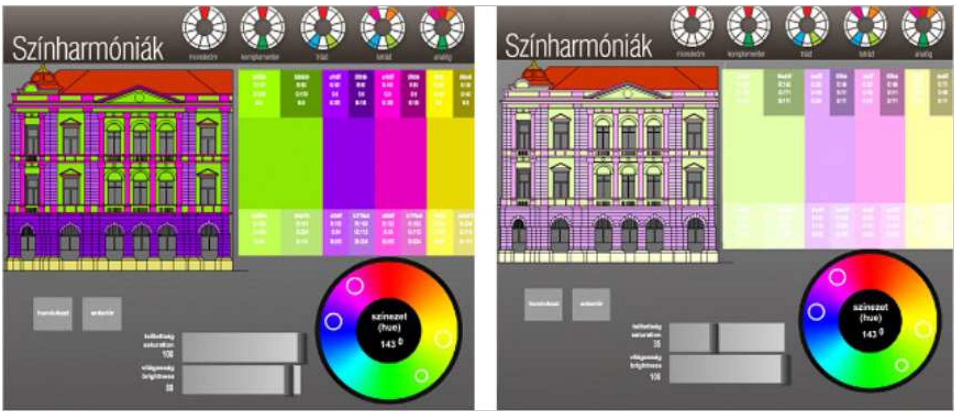

Figure 7: Coloring building exteriors

5.4 Tests

The software provides tests allowing the instructor to monitor students' progress, and adds the option of digital evaluation to assist the process. Using the tests is especially advisable in larger student groups, where it saves time, allows for easier comparisons, a quicker recognition of problematic parts in the taught material, and makes it easier to identify students who progress slower than usual, or on the contrary, the ones who show talent in the field. Question types:

- Single choice and multiple choice questions: this type of test is useful in checking students' knowledge on the theoretical aspects of taught material.

- Showing the answer (problem accompanied by an image): this kind of examination is used to test the ability to recognize, define and distinguish colors.

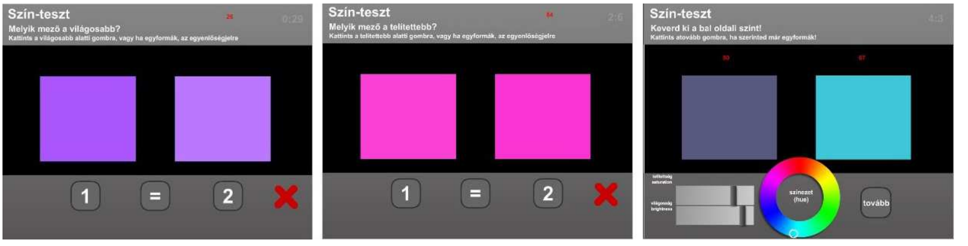

Figure 8: Comparison by brightness and saturation, furthermore color mixing

- Determining the correct order An active type of test to measure how effectively students can put samples with different brightness and saturation in the correct order. It comes at multiple difficulty levels.

Figure 9: Rearranging color samples

Problem: Put the color samples in the correct order of tone. Start with the darkest one, and use as few steps as possible.

Rearrange the displayed samples so that they are positioned from top to bottom on the dark-light scale, and from right to left on the saturated-unsaturated scale. Answer times for each question are monitored as well. In certain problem types, the software only allows the user to progress to the next question if the correct answer has been given, for such problems, the number of tries are monitored instead of answer time.

6 Practical application of the program in education

Benefits of the introduced software are listed below:

- allows students to prepare for exams from the material on their own
- students can carry out basic experiments on their own, acquiring further knowledge to the extent of their personal preference
- material is useful at various skill levels, for students of different grades
- modeling of realistic objects is possible
- contains a large variety of test problems
- useful in examination (side tests in addition to the main exam)

- contains problems at multiple levels of difficulty
- reproduction is possible
- easy to use
- platform-free (works on almost any operation system)
- provides a sense of success to student and instructor alike
- user-friendly

7 Summary, possibilities for improvement

For a civil engineer of the future, it won't be enough to acquire sufficient knowledge of color theory and understanding of how color harmonies are created. It is essential that colleges train professionals who are able to put the studied theory into practice.

Students were looking forward to using a newly designed program to study color theory. After being introduced to the basic structure of the software and the included problem types, they could vote whether they wanted to use the program in classes beside Adobe Photoshop. All students voted on using it, the feature to instantly change color patterns on modeled objects and the variety of options for mixing colors had them convinced. They eagerly experiment with the program even in their free time to practice and gain more experience. Students find the program easy to use and feature-rich.

The first phase of the design process has been completed, but testing is still in progress. If the product turns out to be successful, we plan to implement more features. The software's structure leaves room for more chapters and improvements. We are also planning to release an English version of the educational software.

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Teaching of Binomial Confidence Intervals

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Abstract. In basic statistical textbooks the asymptotic Wald interval and the exact Clopper-Pearson interval are presented for the interval estimating of the binomial proportion. The Wald interval performs poorly and it should not be used. In this paper we recommend to teach the asymptotic Agresti-Coull interval. The Agresti-Coull interval is simple to compute and understand and its derivation is natural for students. This interval performs better compare to the Wald interval. The confidence intervals are illustrated on the example.

Keywords: Binomial distribution, binomial proportion, confidence interval.

Classification: K60.

1 Introduction

The interval estimation of a binomial proportion is one of the basic problem of statistics. The binomial distribution is used in statistics in various application. In general, the random variable X has binomial distribution, if X represents the number of successes in n independent Bernoulli trials. The probability of a success is the same for each trial and is denoted by π.

Let random variable X follows a binomial distribution with parameters $n \in \mathcal{N}$ and $\pi \in (0, 1)$. The probability that a random variable X is equal to the value x is given by

$$P(X = x) = \binom{n}{x} \pi^x (1-\pi)^{n-x}, \quad x = 0, 1, ..., n. \tag{1}$$

The parameter π is also called binomial proportion. In practice a value of the parameter π is usually unknown and must be estimated. The maximum likelihood estimator for the parameter π from the sample is

$$p = \frac{x}{n}, \tag{2}$$

where x is a number of successes in a random sample of size n. We are interested in interval estimating of the parameter π.

The $100.(1-\alpha)\,\%$ two-sided confidence interval for parameter π is interval $CI(x, n) = \langle p_L, p_U \rangle$ such as

$$P\left(p_L \le \pi \le p_U\right) \ge 1 - \alpha, \tag{3}$$

where $(1-\alpha)$ is the desired confidence coefficient and $\alpha \in (0, 1)$.

The literature contains several methods for constructing confidence intervals for the binomial proportion. In basic statistical textbooks there are presented the Wald interval, Laplace (1812) and the Clopper-Pearson interval, Clopper and Pearson (1934). The Clopper-Pearson interval is based on exact binomial distribution. The Wald interval is based on the standard normal approximation to the binomial distribution. It is known that the Wald interval performs very poorly in term of the coverage probability. According to, this interval should not be used, Newcombe (1998), Brown and etc. (2001), Pires and Amado (2008). Instead the Wald interval we recommend to teach the Agresti-Coull interval, Agresti and Coull (1998). Its derivation is similar to the Wald interval. This interval performs very well and is simply to compute and to understand for students.

2 Alternatives of confidence intervals

The Clopper-Pearson interval. The Clopper-Pearson interval is based on exact binomial distribution. This interval is often called "exact" interval. For given $X = x$, $0 < x < n$, lower and upper bounds of the $100.(1-\alpha)\,\%$ Clopper-Pearson interval are

$$\begin{aligned} p_L &= \frac{x}{x + (n-x+1)\,F_{1-\frac{\alpha}{2}}\,(2(n-x+1),\,2\,x))}, \\ p_U &= \frac{(x+1)\,F_{1-\frac{\alpha}{2}}\,(2(x+1),\,2(n-x))}{n-x+(x+1)\,F_{1-\frac{\alpha}{2}}\,(2(x+1),\,2(n-x))}, \end{aligned} \tag{4}$$

where $F_\alpha(r,\,s)$ is α-quantile of F-distribution $F(r,s)$. If $x = 0$ then $p_L = 0$ and $p_U = 1 - \left(\frac{\alpha}{2}\right)^{\frac{1}{n}}$. If $x = n$ then $p_U = 1$ and $p_L = \left(\frac{\alpha}{2}\right)^{\frac{1}{n}}$.

The Wald interval. The Wald interval is based on the standard normal approximation to the binomial distribution. The random variable

$$U = (p-\pi)/\sqrt{\frac{p\,(1-p)}{n}} \tag{5}$$

is approximately standard normally distributed. Thus for given $X = x$ lower and upper bounds of the $100.(1-\alpha)\,\%$ Wald interval are

$$p_L = p - k_{1-\frac{\alpha}{2}}\sqrt{\frac{p\,(1-p)}{n}}, \qquad p_U = p + k_{1-\frac{\alpha}{2}}\sqrt{\frac{p\,(1-p)}{n}}, \tag{6}$$

where k_α is $\alpha-$quantile of a standard normal distribution $N(0,\,1)$.

Why are there several methods for constructing the binomial confidence intervals?

To evaluate the performance of the confidence intervals these criteria: coverage probability, conservatism and interval length are important.

The coverage probability. The coverage probability is defined for given $n \in \mathcal{N}$ and $\pi \in (0,\,1)$ as

$$C(n,\,\pi) = \sum_{x=0}^{n} I(x,\,\pi)\,\binom{n}{x}\,\pi^x\,(1-\pi)^{n-x}, \tag{7}$$

where $I(x,\,\pi) = \begin{cases} 1 & \text{if} \quad \pi \in CI(x,n) \\ 0 & \text{if} \quad \pi \notin CI(x,n) \end{cases}$ is an indicator function.

If for given $n \in \mathcal{N}$ and $\pi \in (0,\,1)$ is $C(n,\,\pi) \geq 1-\alpha$, then the confidence interval is conservative.

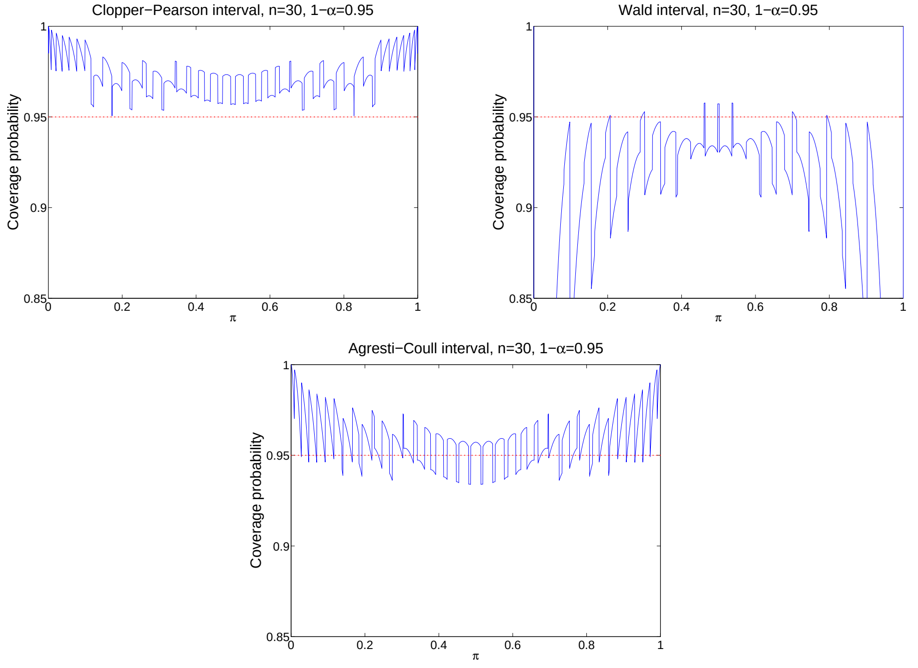

Figure 1: Coverage probability of the 95% confidence intervals for $n = 30$

The coverage probability is probability, that confidence interval contains the parameter π. Due to the discrete nature of the binomial distribution, the coverage probability can not be exactly equal to the nominal level $(1-\alpha)$ at all possible values. We prefer confidence interval with coverage probability near to the nominal level $(1-\alpha)$.

The expected length. The expected length of the confidence interval is defined as

$$EL(n,\,\pi) = \sum_{x=0}^{n} \left[p_U(x,\,n) - p_L(x,\,n)\right] \binom{n}{x} \pi^x \,(1-\pi)^{n-x}\,, \tag{8}$$

where $p_U(x,\,n)$, $p_L(x,\,n)$ are bounds of particular confidence intervals.

We prefer confidence interval with shorter expected length.

The Clopper-Pearson interval is for all $n \in \mathcal{N}$ and $\pi \in (0,\,1)$ conservative interval and too wide (Figure 1, Figure 2). Interval is good choice in the situation, when the coverage probability must be guaranteed to be equal to or above nominal level $(1-\alpha)$ and when the sample size is small.

The Wald interval is the best known and most commonly used interval. The Wald interval is simple and its derivation is natural for students. The textbooks give "safety" recommendation, when may be the Wald interval used, for example if $n\,p \geq 5$, $n\,(1-p) \geq 5$ or $n\,p\,(1-p) \geq 5$ or n is very large. The expected length of the

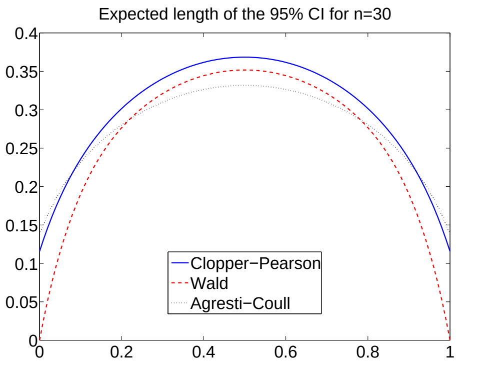

Figure 2: Expected length of the 95% confidence intervals for $n = 30$

Wald interval is short (Figure 2), unfortunately this simple interval suffers from a lot of mistakes. The interval has problem with the zero width interval when $X = 0$ or $X = n$. A lower bound of the interval can be below 0 for X close to 0 and the upper bound can be upper 1 for X close to n. It is known that the Wald interval performs very poorly in term of the coverage probability. The coverage probability is below nominal level $(1-\alpha)$ even for large sample sizes (Figure 1). The Wald interval does not guarantee the good quality of the estimation of the binomial proportion. According to, this interval should not be used, Newcombe (1998), Brown and etc. (2001), Pires and Amado (2008).

The Agresti-Coull interval. Instead the Wald interval we recommend to teach the Agresti-Coull interval, Agresti and Coull (1998). The Agresti-Coull interval is based on the standard normal approximation to the binomial distribution and is similar to the Wald interval. Agresti and Coull (1998) introduced modification of the Wald interval by adding two successes and two failures into sample. The estimator for the parameter π is

$$p_w = \frac{X+2}{n+4}. \tag{9}$$

For given $X = x$ lower and upper bounds of the 100. $(1-\alpha)$ % Agresti-Coull interval are

$$p_L = p_w - k_{1-\frac{\alpha}{2}} \sqrt{\frac{p_w\,(1-p_w)}{n+4}}, \qquad p_U = p_w + k_{1-\frac{\alpha}{2}} \sqrt{\frac{p_w\,(1-p_w)}{n+4}}, \tag{10}$$

where k_α is $\alpha-$quantile of a standard normal distribution $N(0,\,1)$.

This interval has the disadvantage to have the lower bound $p_L < 0$ and the upper bound $p_U > 1$. But never degenerate to interval with the zero width interval as the Wald interval. The Agresti-Coull interval has a good coverage probability, compare to the Wald interval (Figure 1). The Agresti-Coull interval performs very well, it is simply to compute and to understand. Many autors, for instance Brown and etc. (2001), Pires, Amado (2008), recommend the Agresti-Coull interval for presentation and teaching. Brown and etc. (2001) recommend this interval for sample sizes $n \geq 40$. This interval is a good starting point to teach binomial confidence intervals.

2.1 Example

A random sample of 30 consumers taste tested a new food product. 9 consumers like a new food product. We find the 95 % confidence interval to estimate of the proportion of the consumers who like the new food product.

Solution. The point estimator of the proportion of the consumers who like a new food product is

$$p = \frac{x}{n} = \frac{9}{30} = 0,3.$$

- *The Clopper-Pearson interval.* The lower and upper bounds of the interval are

$$p_L = \frac{x}{x + (n - x + 1)\, F_{1-\frac{\alpha}{2}}\,(2(n - x + 1),\, 2\,x))} = \frac{9}{9 + 22.\,2,3673} = 0,147$$

$$p_U = \frac{(x + 1)\, F_{1-\frac{\alpha}{2}}\,(2(x + 1),\, 2(n - x))}{n - x + (x + 1)\, F_{1-\frac{\alpha}{2}}\,(2(x + 1),\, 2(n - x))} = \frac{10.\,2,0499}{30 - 9 + 10.\,2,0499} = 0,494$$

  The 95 % Clopper-Pearson interval is $\langle 0,147;\, 0,494\rangle$.

- *The Wald interval.* The lower and upper bounds of the interval are

$$p_L = p - k_{1-\frac{\alpha}{2}} \sqrt{\frac{p\,(1 - p)}{n}} = 0,3 - 1,96.\sqrt{\frac{0,3.\,0,7}{30}} = 0,136$$

$$p_U = p + k_{1-\frac{\alpha}{2}} \sqrt{\frac{p\,(1 - p)}{n}} = 0,3 + 1,96.\sqrt{\frac{0,3.\,0,7}{30}} = 0,464$$

  The 95 % Wald interval is $\langle 0,136;\, 0,464\rangle$.

- *The Agresti-Coull interval.* The estimator for the parameter π is

$$p_w = \frac{x + 2}{n + 4} = \frac{11}{34} = 0,3235.$$

  Then the lower and upper bounds of the interval are

$$p_L = p_w - k_{1-\frac{\alpha}{2}} \sqrt{\frac{p_w\,(1 - p_w)}{n + 4}} = 0,3235 - 1,96\sqrt{\frac{0,3235.\,0,6765}{34}} = 0,166$$

$$p_U = p_w + k_{1-\frac{\alpha}{2}} \sqrt{\frac{p_w\,(1 - p_w)}{n + 4}} = 0,3235 + 1,96\sqrt{\frac{0,3235.\,0,6765}{34}} = 0,481$$

  The 95 % Agresti-Coull interval is $\langle 0,166;\, 0,481\rangle$.

From this example, the methods give different intervals. The Agresti-Coull interval is the narrowest and the Clopper-Pearson interval is the widest. We prefer the Agresti-Coull interval. We have 95 % confident that the proportion of the consumers who like a new food product is in interval $\langle 0,166;\, 0,481\rangle$.

3 Conclusion

The binomial distribution is encountered in various situation of everyday life. So interval estimation of the binomial proportion is very interesting and important problem. The normal approximation method serves as a simple way to introduce the idea of the confidence interval. The Wald interval is easy to teach and understand for students, it is easy to calculate by hand, but it performs poorly. We recommend to teach the Agresti-Coull interval because it has some better properties compare to the Wald interval and also it is very important to introduce new knowledge into teaching. Then the learning is more useful and interesting for students.

Acknowledgements

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The use of multimedia in the context of e-exams

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Abstract. Web-based mock e-exams have been held for a few years now. There is a number of differences between traditional and e-exams. The possibility of using multimedia in e-exams is one of them. In this article we will present a few examples of multimedia applications used to create tasks and we will explain their meaning.

Keywords: e-exam, multimedia.

Classification: R20.

1 Introduction

1.1 E-matura - First steps

Web-based mock e-exams have been held for a few years now. There is a number of differences between traditional and e-exams. The possibility of using multimedia in e-exams is one of them. In this article we will present a few examples of multimedia applications used to create tasks and we will explain their meaning.

1.2 E-matura - EU project

The e-matura project is another step leading towards the development of exterior exams and e-evaluation. There are no such extensive solutions either in Europe or in the world. By means of this project we intend to show that first attempts of its implementation could be realised in Poland as soon as in four years' time. The decision will depend on the Minister of Education. The first step to implement the e-matura is possible at the so-called extra date of matura.

The e-matura project is a modern and innovative examination system on a countrywide scale, which helps to solve the remaining problems that have appeared while conducting the exams. The system allows conducting matura exams in mathematics along with the computers connected to the Internet. The course of the exam is similar to a regular matura exam in which students sit for it at a fixed time in front of the computers and simultaneously start taking it. When the students begin the exam, they get an access to the exam questions presented with the aid of multimedia. When a student does not understand how to do a given task, he or she can make use of context-based information ascribed to each question.

The target users of e-matura system will be final form students who will be able to use the materials and exams attached to the system for the students to improve their knowledge and prepare them better for the state exam. The system has been prepared to help those students from distant places where access to the Internet is limited (due to frequent connection loss, low bandwidth access) - by means of a 'client group' application. Thanks to that each student on the e-matura platform has equal chances and takes the exam on the same principles regardless of where he or she comes from to take part in the project. Moreover, the system supports the disabled students by adjusting the user's interface to those who are blind.

The exam tasks can be classified as the so-called closed and open-ended. Closed-ended tasks consist of distractors (samples of wrong answers) and one or a few best responses (correct answers). In open-ended tasks the students independently form and write down the answers. Closed-ended tasks are convenient to use when it comes to creating an automatic evaluation system, in case of scanning exam sheets with answers as well as the systems of online exams.

The e-matura project is being made in a versatile way to serve exams in other subjects such as physics or geography as well. The system may also be useful for both teachers and students to facilitate the didactic process. Since the e-matura is an IT system which thanks to advanced algorithmes of marking questions can largely simplify and support the teacher's work so that the students will be able to solve more tasks individually and check their chances at the same time without the necessity that all the exam sheets should be marked by the teacher.

The teacher has also an access to the reports made automatically in the e-matura system and can watch the progress made by a particular student and check where the student has problems and must largely improve his or her knowledge. The application facilitates an extended system of reporting. The system logs not only the standard result but also such data as how often the student entered certain questions or how long it took him or her to solve a given question. Thanks to such information the teachers as well as the people preparing the matura exam can adjust the questions even better to make them clear-cut for the students.

The e-matura project is an innovative approach to the idea of examining students on a large scale using the system based on the Internet. Using the project to conduct the matura exam involves certain requirements concerning dates and hours when such an exam takes place. In order to standardise the rules of exam-taking for all the participants of the project, the system must enable a large number of students to sit for an exam simultaneously. The system has been designed by using dispersed infrastructure in both the data base and the application available for the students.

The data base is a key element of the project which provides an access to the secret questions and the place where the answers given by the students are stored (till the moment that the exam commences). The data base has been created with a database engine of Microsoft SQL Server 2008 R2. A special cluster has been made which consists of two physical database servers connected by means of SAN net to the shared matrix based on hard discs with the SAS interface. The database servers have been physically separated from the Internet and are available only by means of the application provided by the application servers. Using the cluster technology enables high efficiency and safety - in case of a physical failure of one of the servers takes over its function and provides services successfully so that the final user will not even realise that there have been any technical problems. It is necessary that the database engine is highly effective since all the information about the user's activity during the exam is stored in the data base (the answers - even if the user changes the answer, each answer given by him or her is separately registered for the further analysis, the timing of giving answers, how often he or she enters a particular question, the information about using technical text assistance etc.)

2349 students from the schools in the province of Lodz took part in testing the project in April, 2011. Therefore it was possible to measure how overloaded the data base was at the level of about 10-15% of the use of the equipment which was purchased for the purposes of of this project. On the basis of the synthetic tests conducted by means of the servers which carried out controlled DDOS attacks on database servers of the e-matura project it is concluded that the purchased equipment should be sufficient for about 25000-30000 matura students (as compared to the number of first-time matura students in the province of Lodz which amounted to 22315), simultaneous users referring to the data base by means of the e-matura application. Taking into account the results of synthetic test and the constantly implemented optimizations in the system the equipment especially purchased for the purposes of the project should meet the demands of conducting the matura exam for all the matura students from the province of Lodz. In order to increase the number of students, it will be necessary to make a bigger investment in purchasing more equipment.

The e-matura application is a user's interface which the students use to communicate with the database by downloading the questions and answering them. The application has been created on the basis of a 'fat client' model by means of 4.0. Silverlight technology. Applying this kind of model enabled making a much safer application as well as highly increased convenience for the students to use the application in the exam. The application is used by the WWW search engine and from a user's perspective it operates like a website. However, it is an application modelled on a 'fat client', which means that the whole application is downloaded to a user's local computer and works entirely independently. The user's interface is as responsive for the users connected to the Internet with the bandwidth as for those who have lower connections, which would be impossible to achieve using a standard WWW website as the users with a lower connection would wait much longer to browse the sites with subsequent questions. The e-matura application eliminates this problem increasing the equality of chances among all the users taking the exam. The application downloads all the questions at the very beginning and refers to the server only when answering a given question. Even if the Internet connection is broken for a moment, the user's answers are saved in the hand memory of the application and when the connection with the server is regained the application sends all the data in the background without an influence on the user's work.

The physical environment which is used to serve the e-matura application has been created on the basis on four servers by using the Microsoft Windows 2008 R2 software. 11S in 7.5 version is a server that serves the application for the final users and mediates in communication between the application and the database server. Furthermore, there is another server that functions as a "load balancer" which receives all the entries of the users who start the application. This server directs the users' questions to the server opening the application in a way to balance the overloading among the four servers and as a result to maximize the efficiency of serving data. Using the dispersed infrastructure improves the safety of using the application by means of securing against a failure of the equipment. In case of a failure of one of the servers, the questions sent to it are redirected to the remaining servers which automatically take over its role. The e-matura project has been created in an innovative way to meet the demands made for matura exams in mathematics. The project is being made in a way to be the most versatile and applicable after some alterations for other subjects as well.

1.3 E-matura in the context of traditional matura

The system of exterior exams has been used in Poland for more than ten years. The system of exterior exams used in Poland is based on traditional written exams taken by students in paper which are then marked equally traditionally by teachers examiners. IT technologies are not used in these processes. This situation is far from young people's expectations and interests - keen on computers and the Internet. Their sensitivity, aspirations and ambitions are centered on the virtual world.

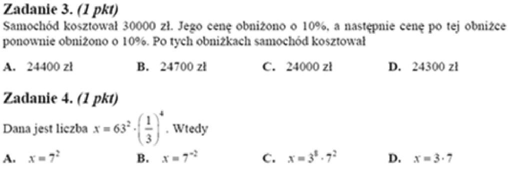
The matura exam in mathematics tests the knowledge and skills defined in the standards of exam requirements. The tested skills include:

- using and creating information,
- using and interpreting the representation,
- mathematical models,
- using and creating strategies
- undersatnding and giving arguments

The exam sheets are prepared by the local exam committees. The Central Examination Committee chooses one of them. The matura exam in mathematics consists of two parts: basic and extended.

The basic part of the exam is compulsory. It lasts 170 minutes. The exam tasks include the requirements for the basic level. The exam sheet for the basic level consists of three groups of tasks:

- Group 1 - consists of 20-30 closed-ended tasks. There are four answers to each task and only one of them is correct. Students receive 0-1 points for each task, Figure 1.



Zadanie 3. ***(1 pkt)***
Samochód kosztował 30000 zł. Jego cenę obniżono o 10%, a następnie cenę po tej obniżce ponownie obniżono o 10%. Po tych obniżkach samochód kosztował

A. 24400 zł **B.** 24700 zł **C.** 24000 zł **D.** 24300 zł

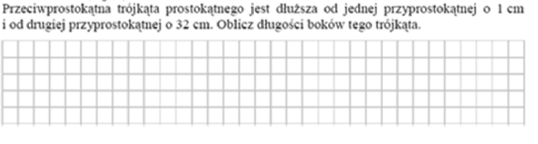
Zadanie 4. ***(1 pkt)***
Dana jest liczba $x = 63^2 \cdot \left(\frac{1}{3}\right)^4$. Wtedy

A. $x = 7^2$ **B.** $x = 7^{-2}$ **C.** $x = 3^8 \cdot 7^2$ **D.** $x = 3 \cdot 7$

Figure 1: Sample task for group 1, source: CEC

  The student marks the answer on the answer sheet.

- Group 2 - consists of 5-10 short answer open-ended tasks scored 0-2 for each question, Figure 2.



Zadanie 28. ***(2 pkt)***
Przeciwprostokątna trójkąta prostokątnego jest dłuższa od jednej przyprostokątnej o 1 cm i od drugiej przyprostokątnej o 32 cm. Oblicz długości boków tego trójkąta.

Figure 2: Sample task for group 2, source: CEC

- Group 3 - consists of 3-5 extended answer open-ended tasks scored 0-5 (depending on the type of the task), Figure 3.

Zadanie 32. ***(4 pkt)***
Ciąg $(1, x, y-1)$ jest arytmetyczny, natomiast ciąg $(x, y, 12)$ jest geometryczny. Oblicz x oraz y i podaj ten ciąg geometryczny.

Figure 3: Sample task for group 3, source: CEC

The student can score 50 points at the maximum for solving all the tasks.

The extended matura exam in mathematics last 180 minutes. It consists of about 12 open-ended tasks. The tasks concern mathematical problems, Figure 4.

Zadanie 1. ***(4 pkt)***
Uzasadnij, że dla każdej liczby całkowitej k liczba $k^6 - 2k^4 + k^2$ jest podzielna przez 36.

Figure 4: Sample task for the extended part, source: CEC

Evaluation:

Open-ended tasks are marked and evaluated by the examiners are maths teachers and those who completed courses for examiners and hold a certificate issued by the Head of the Central Examination Committee. Students mark the answers to the closed-ended tasks on a special answer sheet. The solutions to the open-ended tasks are marked on the basis of detailed criteria of evaluation and unified throughout the country.

The answer sheet is detached from the the exam paper and inserted into a special reader.

At present the most modern trend of examining is mainly to replace the paper form for an electric one. These changes pose new challenges for example the possibility of multimedia to create tasks. Sample tasks with the aid of multimedia will be presented in the next chapter.

The Central Examination Committee began the realisation of the E-evaluation project in 2007. The basic difference between traditional evaluation and e-evaluation is that the examiners do not receive the exam papers to mark in paper but in the form of a picture. The system of e-evaluationhas been introduced in many countries like Great Britain and the USA. The research done on e-evaluation has shown many of its advantages. The next step to develop examining and evaluating in Poland will become e-exams.

2 Multimedia-based task construction

Sample tasks:
Task 1. The value of the rhomboid surface:
A. increasing
B. decreasing

C. constant
D. undefined

The description of the animation: the rhomboid moves without changing the lengths of the parallel sides. The length of the height does not change either.

Task 31. (category 1) the surface of the rhomboid is:
A. increasing
B. decreasing
C. constant
D. undefined

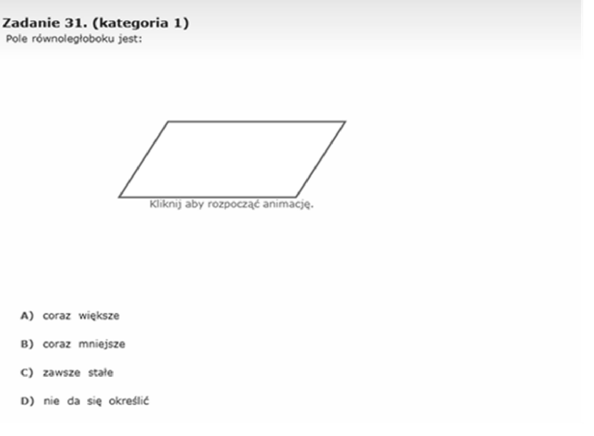


Figure 5

Task 2: the angle of vertex C
A. increasing
B. decreasing
C. constant
D. undefined
The description of the animation: point C moves around the circle, see figure 6.

Task 3. There is a triangle ABC. The surface of the trangle:
A. increasing
B. decreasing
C. constant
D. undefined

The description of the animation: point C moves along a straight line which is parallel to the line with points A and B.

Task 4. There are four subsequent figures in figure 4. Find out how new figures are

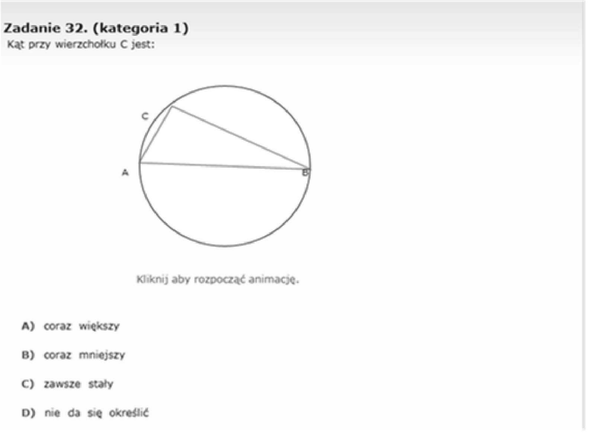


Figure 6

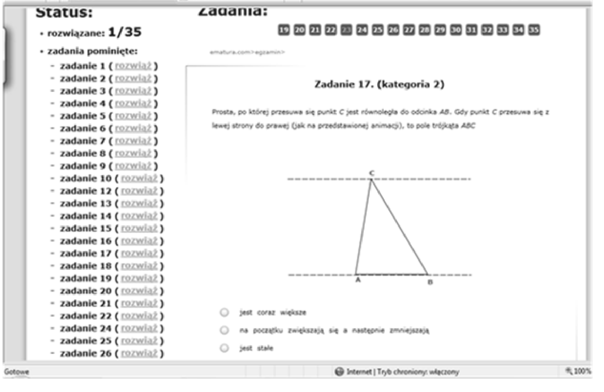


Figure 7

created and draw the tenth one. Fill in the formulas on the appropriate surfaces.

Description: the student can draw segments. The systems checks the accuracy of the drawn segments.

Task 9. In the figure there are six subsequent figures. Find out how new figures are created and answer the questions. P_n stands for the surface area of the n-th figure $(n = 1, 2, 3 \ldots)$.
Define the formula.
Define the difference.
Finish drawing Figure 10.

3 Conclusion

Target replacement of exams with e-exams should increase the students' interest in education at the same time e-exams will contribute to creating an IT-oriented society. One may expect that students' psychological barrier towards IT education

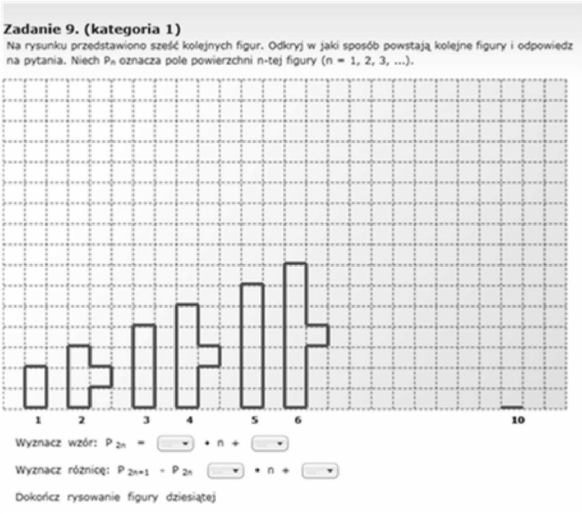


Figure 8

will lessen thanks to e-exams, which should increase the number of students at Technical Universities. The e-matura in mathematics is a pioneer project. Using multimedia to construct tasks creates new areas for scientific research. Multimedia may become a key element of modern exams and evaluation.

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Attitudes of pupils and teachers as important factor for mathematics education

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Abstract. The paper deals with pupils' attitudes towards mathematics, their structure and changes between the fifth and the ninth grade of primary school. We discuss statistically significant factors influencing the attitudes towards mathematics and the impact on the education. Important factor is the teacher and the teaching learning styles. Therefore, in the paper we discuss also teachers' attitudes and their influence on the education.

Keywords: mathematics education, attitudes towards mathematics, emotion, beliefs.

Classification: C20; C70.

1 Introduction

The importance of the pupils' attitudes towards mathematics is supported by the opinion, believed to be true in scientific and teacher communities which states that pupils learn more effectively and they are more interested in the mathematics lesson and are performing better if they have positive attitudes towards mathematics (Ma & Kishor, 1997).

In this paper we deal with the result of the questionnaire survey on pupils attitudes towards mathematics performed in Slovakia. Used questionnaire was created during the study on the structure of pupils' attitudes system (De Corte & Op't Eynde, 2002). In the later experiments the questionnaire was modified to find out the compatibility of its use in various European states, in concrete in England, Spain and Slovakia (Andrews et all, 2007; Andrews et all, 2008). The research was done in three phases in Slovakia.

2007	76 pupils in the 5th class (9-11 years old) and 128 pupils in the 9th class (15-16)	The aim was to create version of questionnaire compatible with questionnaires in England and in Spain and to investigate development of pupils' mathematics attitudes during school education.
2008	241 pupils in the 9th class (14-16)	The aim was to improve the questionnaire.
2009	746 pupils in the 8th class (13-15)	The research was focused on the links between various aspects of pupils' attitudes.

The results of researches in Slovakia were already partially published (Andrews et all, 2007, 2008; Vankúš & Kubicová, 2010). In this paper we report more closely on the part of the results from the survey carried out in 2007 on a sample of 204 Slovak pupils, 76 in the 5th year (9-11 years old) and 128 pupils in the 9th year of the primary school (14-16 years old).

2 Questionnaire survey of pupils' mathematics attitudes

The survey was performed at the primary school Alexandra Dubeka in Bratislava. The first page of questionnaire gave information about questionnaire and its goals. It also included items about pupils' sex and age. The other items of questionnaire were Likert type, with six possibilities Agree Strongly (1), Agree (2), Agree Slightly (3), Disagree Slightly (4), Disagree (5), Disagree Strongly (6). Questionnaire had 60 items those were created to investigate following areas:

- Positive or negative attitudes: Attitudes towards mathematics as school subject and its teaching and self-evaluation of mathematics abilities.
- Passive learning or active learning: This area was studying pupils' beliefs on the nature of school mathematics.
- Teacher's influences on pupils' attitudes.

Distribution of items in the areas shows Table 1.

Table 1: Distribution of questionnaire items

Attitudes towards mathematics as school subject and its teaching and self-evaluation of mathematics abilities	
Positive	3. Everyone can learn mathematics.
	4. Mathematics enables us to better understand the world we live in.
	7. Mathematics is used all the time in people's daily life.
	8. I can use what I learn in mathematics in other subjects.
	9. I think I will do well in mathematics this year.
	10. By doing the best I can in mathematics I try to show my teacher that I'm better than other students.
	11. I like doing mathematics.
	12. I expect to do well on the mathematics tests and assessments we do.
	13. I try hard in mathematics to show the teacher and my fellow students how good I am.
	14. I understand everything we have done in mathematics this year.
	15. I think mathematics is an important subject.
	17. I can understand even the most difficult topics taught me in mathematics.
	20. Im very interested in mathematics.
	25. I don't have to try too hard to understand mathematics.
	26. I can usually do mathematics problems that take a long time to complete.
	27. I find I can do hard mathematics problems with patience.
	33. I study mathematics because I know how useful it is.
	34. Knowing mathematics will help me earn a living.
	35. Mathematics is a worthwhile and necessary subject.
	39. I think that what I am learning in this class is useful for me to know.
	40. I like what I am learning in this class.
	41. I think that what I am learning in this class is interesting.
	43. I am certain I can learn how to solve the most difficult mathematics problem.
	44. Compared with others in the class, I think I'm good at mathematics.

Negative	18. My only interest in mathematics is getting a good grade.
	21. If I cannot do a mathematics problem in a few minutes, I probably cannot do it at all.
	22. If I cannot solve a mathematics problem quickly, I quit trying.
	23. Only very intelligent students can understand mathematics.
	24. Ordinary students cannot understand mathematics, but only memorise the rules they learn.
	36. Mathematics has no relevance to my life.
	37. Studying mathematics is a waste of time.

Pupils' beliefs on the nature of school mathematics	
Passive learning	1. Mathematics learning is mainly about having a good memory.
	2. It's a waste of time when our teacher makes us think on our own.
	6. There is only one way to find the correct solution to a mathematics problem.
	29. Getting the right answer is more important than understanding why the answer works.
	38. Only the mathematics to be tested is worth learning.
Active learning	5. Everybody has to think hard to solve a mathematics problem.
	16. I prefer mathematics when I have to work hard to find a solution.
	19. If I try hard enough I understand the mathematics we are taught.
	28. Routine exercises are very important in the learning of mathematics.
	30. Time used to understand why a solution works is time well spent.
	31. Discussing different solutions to a mathematics problem is a good way of learning mathematics.
	32. I think it is important to learn different strategies for solving the same problem.
	42. I prefer class work that is challenging so I can learn new things.

Teacher's influences on pupils' attitudes	
Positive	45. My teacher thinks mistakes are okay as long as we are learning from them.
	46. My teacher explains why mathematics is important.
	47. My teacher always shows us, step by step, how to solve a problem, before giving us exercises.
	48. My teacher really wants us to enjoy learning new things.
	49. My teacher understands our problems and difficulties with mathematics.
	50. My teacher listens carefully to what we say.
	51. We do a lot of group work in this mathematics class.
	52. My teacher always gives us time to explore new problems and try out different solution strategies.
	53. My teacher appreciates it when we try hard, even if our results are not so good.
	54. My teacher is friendly to us.
	55. My teacher tries to make the mathematics lessons interesting.
	57. My teacher wants us to understand the content of this mathematics course.

Negative	56. My teacher thinks she/he knows everything best.
	58. My teacher wants us just to memorise the content of this mathematics course.
	59. My teacher does not really care how we feel in class.
	60. My teacher is too absorbed in mathematics to notice us.

The results of the survey (average score in the items) are shown in the following figures. The lower values show more strong agreement with item. In the area of positive or negative attitudes towards mathematics as school subject and its teaching and self-evaluation of mathematics abilities were included 24 items.

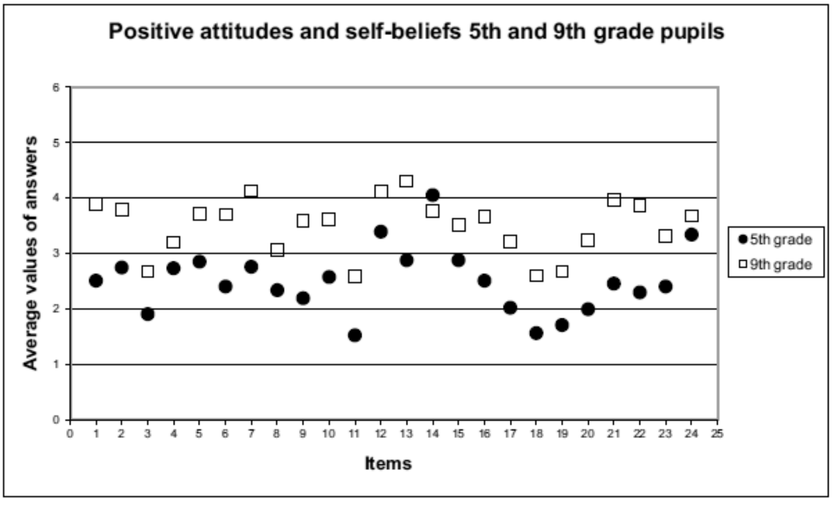

Figure 1: Positive attitudes and self-beliefs of 5th and 9th grade pupils

Items:

1. Everyone can learn mathematics.
2. Mathematics enables us to better understand the world we live in.
3. Mathematics is used all the time in people's daily life.
4. I can use what I learn in mathematics in other subjects.
5. I think I will do well in mathematics this year.
6. By doing the best I can in mathematics I try to show my teacher that I'm better than other students.
7. I like doing mathematics.
8. I expect to do well on the mathematics tests and assessments we do.
9. I try hard in mathematics to show the teacher and my fellow students how good I am.
10. I understand everything we have done in mathematics this year.
11. I think mathematics is an important subject.
12. I can understand even the most difficult topics taught me in mathematics.

13. I'm very interested in mathematics.
14. I don't have to try too hard to understand mathematics.
15. I can usually do mathematics problems that take a long time to complete.
16. I find I can do hard mathematics problems with patience.
17. I study mathematics because I know how useful it is.
18. Knowing mathematics will help me earn a living.
19. Mathematics is a worthwhile and necessary subject.
20. I think that what I am learning in this class is useful for me to know.
21. I like what I am learning in this class.
22. I think that what I am learning in this class is interesting.
23. I am certain I can learn how to solve the most difficult mathematics problem.
24. Compared with others in the class, I think I'm good at mathematics.

The items of this figure we can further divide to more detail specific areas. The items 1, 7, 13, 21 and 22 inform us about the positive attitudes of pupils towards mathematics and the mathematics knowledge in that year of study. From the average values of answers for the 9th grade we can see that they have much worse attitudes towards mathematics as the 5th grade. It could be influenced by the fact that for the 9th grade students is mathematics usually more abstract and for some pupils therefore less interesting. The items 2, 3, 4, 11, 17, 18, 19 and 20 are about awareness of importance of mathematics. From the Figure 1 we can see that both the 5th grade and the 9th grade pupils are aware about importance of mathematics. The 9th grade is probably also influenced by the fact that mathematics is needed for their further mathematics education at the high school. The positive self-beliefs are checked by items 5, 8, 10, 12, 14, 15, 16, 23 and 24. In those the 5th grade students showed positive mathematics self-beliefs. Pupils of the 9th grade were to their mathematics abilities more critical. In the items 6 and 9 we can see influence of the outer motivational impetus. This is more significant for the 5th grade students. This is due to the pupil's development and its different level between the 5th grade and the 9th grade pupils. Teacher as person that pupils want to be in favour of them is more important for the 5th grade as for the 9th grade pupils.

Items:

1. My only interest in mathematics is getting a good grade.
2. If I cannot do a mathematics problem in a few minutes, I probably cannot do it at all.
3. If I cannot solve a mathematics problem quickly, I quit trying.
4. Only very intelligent students can understand mathematics.
5. Ordinary students cannot understand mathematics, but only memorise the rules they learn.
6. Mathematics has no relevance to my life.
7. Studying mathematics is a waste of time.

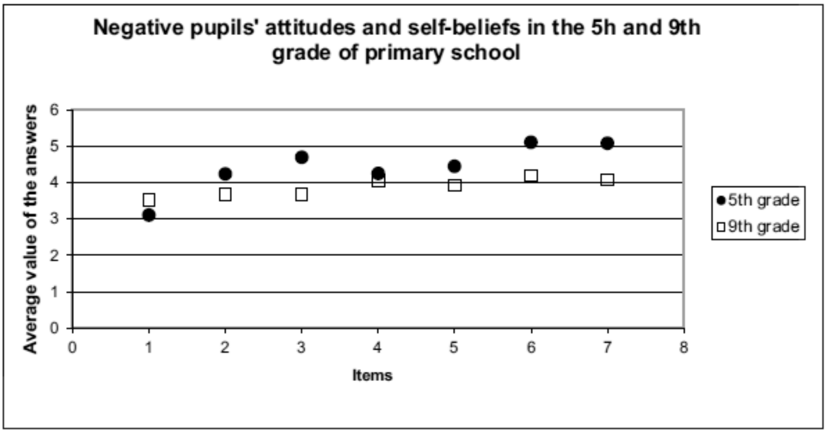

Figure 2: Negative pupils' attitudes and self-beliefs

Negative attitudes and self-beliefs appeared more at the 9th grade students. The 5th grade students have stronger link between grades and interest in mathematics. In the area passive learning we have 5 items in the questionnaire. Comparison of the results of the 5th grade students and the 9th grade students is in the Figure 3.

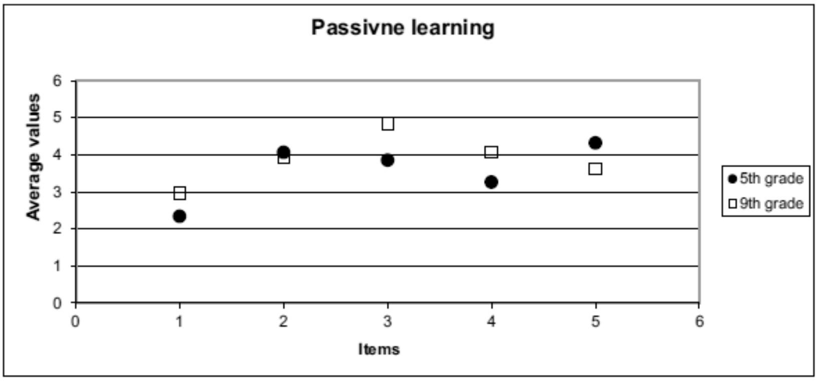

Figure 3: Passive learning of the 5th grade and 9th grade students

Items:

1. Mathematics learning is mainly about having a good memory.
2. It's a waste of time when our teacher makes us think on our own.
3. There is only one way to find the correct solution to a mathematics problem.
4. Getting the right answer is more important than understanding why the answer works.
5. Only the mathematics to be tested is worth learning.

From the figure we can see that both grades students know that mathematics is not about passive learning of information. Also we can see that the 5th grade students agree more with opinion that learning mathematics is more about having good memory, which is connected with their cognitive development. So the 9th grade students have better knowing that mathematics is supposed to improve thinking and that is important to understand the taught stuff. Opposite to passive learning is area of active learning, comparison of the 5th grade students and the 9th grade students is in the Figure 4.

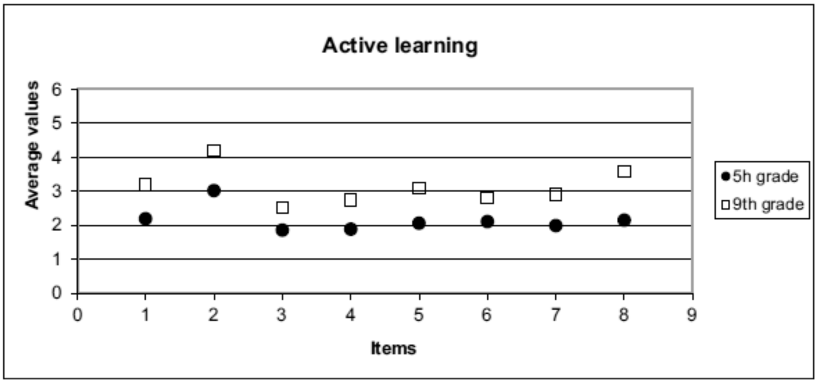

Figure 4: Active learning

Items:

1. Everybody has to think hard to solve a mathematics problem.
2. I prefer mathematics when I have to work hard to find a solution.
3. If I try hard enough I understand the mathematics we are taught.
4. Routine exercises are very important in the learning of mathematics.
5. Time used to understand why a solution works is time well spent.
6. Discussing different solutions to a mathematics problem is a good way of learning mathematics.
7. I think it is important to learn different strategies for solving the same problem.
8. I prefer class work that is challenging so I can learn new things.

The Figure 4 shows that the 5th grade students have bigger inclination towards active learning than the 9th grade students. The 9th grade students have less interest in solving more difficult tasks. In the next area we investigate influence of teacher on the pupils' attitudes.

Items:

1. My teacher thinks mistakes are okay as long as we are learning from them.
2. My teacher explains why mathematics is important.
3. My teacher always shows us, step by step, how to solve a problem, before giving us exercises.

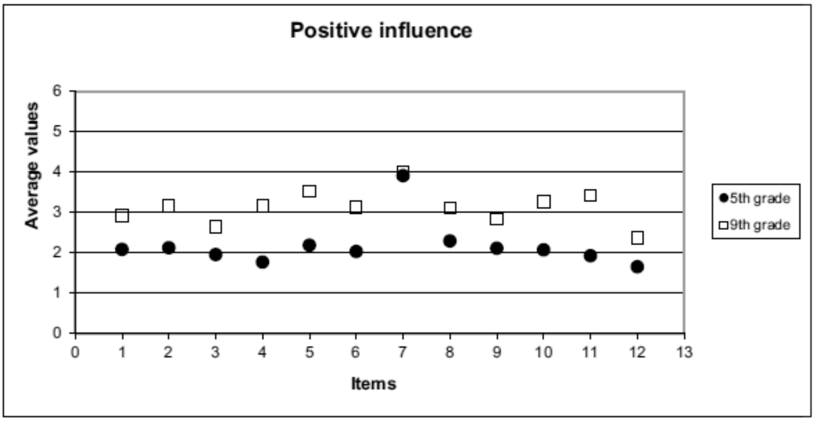

Figure 5: Positive influence of the teacher on pupils' attitudes

4. My teacher really wants us to enjoy learning new things.
5. My teacher understands our problems and difficulties with mathematics.
6. My teacher listens carefully to what we say.
7. We do a lot of group work in this mathematics class.
8. My teacher always gives us time to explore new problems and try out different solution strategies.
9. My teacher appreciates it when we try hard, even if our results are not so good.
10. My teacher is friendly to us.
11. My teacher tries to make the mathematics lessons interesting.
12. My teacher wants us to understand the content of this mathematics course.

Positive influences of the teacher are more felt by the 5th grade students. They have more feeling that teacher is friendly, kind and that he/she wants to make the lessons pleasant for them. The 9th grade students have more feelings of not being understood by the teacher that is connected with their psychological state in the pubescent.

Items:

1. My teacher thinks she/he knows everything best.
2. My teacher wants us just to memorise the content of this mathematics course.
3. My teacher does not really care how we feel in class.
4. My teacher is too absorbed in mathematics to notice us.

From the Figure 6 we can derive that pupils do not think that teachers ignores them. Although, the 9th grade students have more strong feeling that teachers is not paying attention to their feelings on the mathematics lessons. Differences between perception of the 5th grade and 9th grade students are probably due to the teacher's different behaviour towards the different grade students.

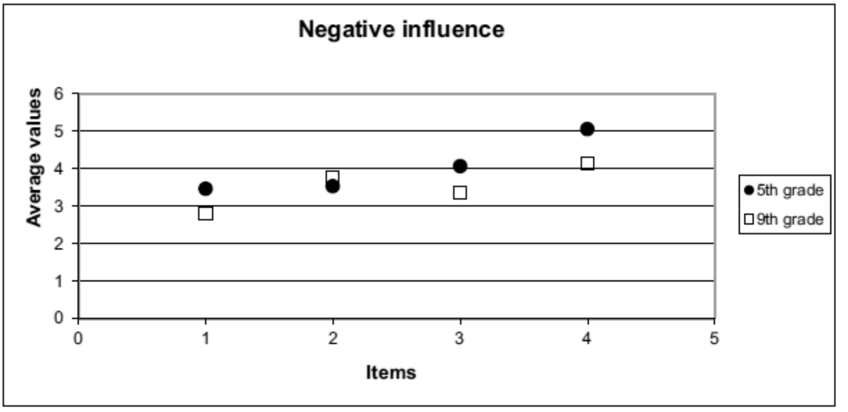

Figure 6: Negative influence of the teacher on pupils

3 Discussion

The questionnaire shows that the 5th grade students in the research sample have more positive attitudes towards mathematics than the 9th grade students. Both the 5th and the 9th grade students realize that mathematics is useful subject. The 9th grade students are more critical towards their mathematics abilities. The 5th grade students are more motivated for studying mathematics by the grades and the evaluation from their peers and teacher. The extrinsic motivation has less influence on the 9th grade students. Both grade students know that mathematics is not about passive acceptation of the knowledge. The 5th grade students show more activity when solving mathematical problems and they have stronger need to find more ways of solving the problems and they take the activity in class as greater challenge. The 5th grade students feel the positive approach of the teacher more than the 9th grade students that can be connected with biological and psychological state of development of the 9th grade students. They are more critical towards teacher and they have stronger feelings of misunderstanding from the teacher.

4 Conclusion

The paper is discussing comparison of the attitudes towards mathematics of the 5th grade students (911 years old) and 9th grade students (1416 years old). The questionnaire survey was carried out in 2007 on the sample of 204 Slovak pupils, 76 in the 5th grade and 128 pupils in the 9th grade. In generally we can say that the questionnaire shows that the 5th grade students in the research sample have more positive attitudes towards mathematics than the 9th grade students. They have more positive self-beliefs in their mathematics knowledge and they feel more positive influences from the mathematics teacher. This can be connected with different approach of teacher towards 9th grade students, different style of their teaching and also with different biological and psychological state of development of the 5th grade and 9th grade students.

These results are in the coherence with previous studies of this subject those have found that the attitudes towards mathematics are often getting worse during the school attendance (Ma & Kishor, 1997). The inadequate difficult tasks, too fast pace of learning and negative influences of teacher have bad impact on pupils' mathematics attitudes and their development during school attendance (Philippou a Christou, 1998). Those school years are very important for the forming of the mathematics attitudes therefore is the teachers' responsibility to positively influence the mathematics attitudes of the pupils (Ma & Kishor, 1997).

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