XXIV CZECH-POLISH-SLOVAK MATHEMATICAL CONFERENCE

Ružomberok, June 6 – 9, 2017 · **Catholic University in Ružomberok**

m czech polish slovak **conference**
2017

PROGRAMME · WEDNESDAY · June 7, 2017

8^{00}	·	9^{30}	**Registration** (Room A201)
9^{30}	·	9^{45}	**Opening ceremony** (Room A244)
9^{45}	·	10^{45}	**Plenary lecture** (Room A244 · Chair Martin Papčo)

PAVOL ZLATOŠ – *Hilbert's Program and Gödel's Incompleteness Theorems*

10^{45}	·	11^{15}	**Coffee Break** (Room A203)
11^{15}	·	12^{40}	**Contributions** (Room A244 · Chair Marián Trenkler)

Murat Sadiku, Nimete Berisha – *A Method on Accelerating Convergence of Trigonometric Series*

Bożena Woźna-Szcześniak, Ireneusz Szcześniak – *Real Time Commitment Logic interpreted over Duration Communication Interpreted System*

Martin Papčo – *Why should we use indicator functions?*

Mateusz Woronowicz, Ryszard R. Andruszkiewicz – *On the square subgroup of an abelian group*

13^{00}	·	14^{00}	**Lunch** (University lunchroom, first floor)
14^{30}	·	15^{30}	**Plenary lecture** (Room A244 · Chair Mária Jurečková)

ROMAN FRIČ – *A dialogue/monologue on probability*

15^{30}	·	16^{30}	**Contributions** (Room A244 · Chair Mária Jurečková)

Adam Płocki – *Probability around us, or what, how and why is mathematized*

		Luljeta Sadiku, Murat Sadiku, Izet Zeqiri – *A Mathematical Model for Financing the Higher Education*
		Izet Zeqiri, Murat Sadiku, Luljeta Sadiku – *An Econometric Model for estimating the effects of financing the Higher Education in the Republic of Macedonia*
16^{30} · 17^{00}		**Coffee Break** (Room A203)
17^{00} · 19^{00}		**Contributions** (Room A244 · Chair Roman Frič)
		Péter Körtesi – *Hand held technology in Mathematics*
		Artur Gola, Lidia Stępień, Marcin Ryszard Stępień, Marcin Ziółkowski – *Applications of Python programs in solving of equations based on selected numerical methods*
		Jiří Přibyl, Petr Eisenmann – *Heuristic Strategies in Graph Theory Problem Solving*
		Zuzana Václavíková – *LARPing the Math*
		Qianjun Tang, Jitka Laitochová – *How to Integrate ICT into Mathematics Teaching?*
		Ingrid Nagyová – *Didactics of mathematics versus didactics of informatics*
19^{30}		**Reception** (University foyer, first floor)

PROGRAMME · THURSDAY · June 8, 2017

8^{30} · 9^{30}		**Plenary lecture** (Room A244 · Chair Petra Konečná)
		Jana KRAJČIOVÁ – *Rapsodické skúsenosti učiteľky matematiky*
9^{30} · 10^{30}		**Contributions** (Room A244 · Chair Petra Konečná)
		Mária Jurečková – *Educational research of the mathematical competence*
		Lucia Csachová – *Real-life problems and financial literacy in Slovak nation-wide testing T9*
		Ján Gunčaga – *Real-life problems in mathematics education*
10^{30} · 11^{00}		**Coffee Break** (Room A203)

11^{00} · 12^{20}	**Contributions** (Room A244 · Chair Adam Płocki)

Antonín Jančařík – *Mají budoucí učitelé matematiky umět počítat?*
Jitka Hodaňová – *Graph and Diagram Using in Mathematics Teaching in Elementary School*
Katarína Sátorvá, Daniela Guffová – *Argumentation and reasoning of prospective teachers*
Petra Konečná, Jakub Poruba – *The state of pre-gradual preparation of Math teachers at the time of the end of existence of branches of study*

13^{00} · 14^{00} **Lunch** (University lunchroom, first floor)

14^{30} · 15^{30} **Plenary lecture** (Room A244 · Chair Antonín Jančařík)

EDITA PARTOVÁ – *Dilema medzi presnosťou a zrozumiteľnosťou vo vyučovaní matematiky*

15^{30} · 16^{00} **Coffee Break** (Room A203)

16^{00} · 18^{30} **Individual trips**

18^{30} **Supper**

PUBLICATION POSSIBILITIES (deadline submission July 31, 2017)

- **Scientific Issues of Jan Długosz University in Częstochowa** ⟨www.scientificissues.ajd.czest.pl⟩
  (Mathematics and its applications, Mathematics education)

- **Communications in Mathematics** ⟨http://cm.osu.cz⟩
  (Mathematics and its applications)

- **Annales Universitatis Paedagogicae Cracoviensis, Studia ad Didacticam Mathematicae Pertinentia**
  ⟨didacticammath.up.krakow.pl⟩ (Mathematics education)

- **Informatics in Education** ⟨www.mii.lt/informatics_in_education⟩
  (ICT in Mathematics education)

Lucia Csachová ⟨lucia.csachova@gmail.com⟩ · **Catholic University in Ružomberok**

Real-life problems and financial literacy in Slovak nation-wide testing T9

The application of mathematics for solution of the real-life problems and the financial literacy are currently highly discussed topics not only in the Slovak school system. Their importance and necessity are emphasized for all levels ISCED 0 – ISCED 3. In annual testing of 15-years-old pupils' mathematical competencies in Slovakia, pupils solve a set of test tasks, and half of them are formulated with a real-life context. From these tasks, we focus on a group whose content is related to financial literacy and on their qualitative analysis.

Roman Frič ⟨fric@saske.sk⟩ · **Mathematical Institute, Slovak Academy of Sciences**

A dialogue/monologue on probability

Since the publication of "Grundbegriffe der Wahrscheinlichkeitrechnung" by A. N. Kolmogorov in 1933 (see the 6th Hilbert's problem), mathematics and probability theory underwent a considerable development. Formulating questions and providing (?) answers (dialogue), I hope to present (monologue) a discourse on probability, biased by my research and pedagogical experience. Q: What is wrong with the classical model of Kolmogorov? First, using a simple example, I hope to explain how "division of random events" leads to a quantum/fuzzy model based on multivalued Łukasiewicz logic. Second, using elementary category theory, it can be shown that the quantum/fuzzy model is "a minimal extension" of the model of Kolmogorov. The extension is similar (?) to the transition from whole numbers to rational and real numbers.

Artur Gola ⟨a.gola@ajd.czest.pl⟩ · Lidia Stępień · Marcin Ryszard Stępień · Marcin Ziółkowski · Jan Długosz University in Częstochowa

Applications of Python programs in solving of equations based on selected numerical methods

From the mathematical point of view, it is always very important to find the exact solution of a given equation. Unfortunately, the effective algorithms exist only for a small group of equations. For example, we can solve only polynomial equations whose degree is less than five. But even in this case, the obtained formulae may be very complicated and they cannot be used in practice, especially in the case of third or fourth degree polynomial equations. For the other groups of equations, that contain in addition logarithmic, exponential or trigonometric functions, it is usually impossible to find exact solutions. In such situations, we use numerical methods to find the approximations of the solutions. Of course, almost all numerical methods have their own limitations that relate to the properties of the function appearing in the equation. Moreover, we usually must know in which intervals the roots of a given equation are. On the other hand, we luckily may use many different methods in the process of finding the approximations. The most important methods are: bisection, secants, tangents and iteration. Computations that are made in this case are also very laborious because of many loops appearing. Therefore, we should use computer programs. One of the most effective programming language that let write short and simple applications that solve problems is Python language. This language additionally has no limitation of the introduced numbers. In this talk we present the mathematical background of the four most used numerical methods of solving equations and we present the examples of Python applications that find the approximations of the roots of the given equations. We also compare the exact and approximate solutions of the polynomial equations of the third or fourth degree. Exact solutions are obtained with the usage of Cardano and Ferrari formulae by the help of Mathematica environment, the approximate ones – based on the selected numerical methods by the help of Python applications.

Ján Gunčaga ⟨fric@saske.sk⟩ · Catholic University in Ružomberok

Real-life problems in mathematics education

The focus of many international studies such PISA has been less about what students know after studying a particular curriculum, and more on students' ability to use what they have learned at school to address authentic, real-life challenges and problems. These studies analyse pupils´ abilities from different points of view, e.g. Quantity; Uncertainty and data; Change and relationships; Space and shape. We present some historical real-life mathematic problems, and some from international studies, too.

Jitka Hodaňová ⟨jitka.hodanova@upol.cz⟩ · **Palacký University in Olomouc**

Graph and Diagram Using in Mathematics Teaching in Elementary School
In the elementary school children learn to orientate themselves in plane, they count objects in the given set (group), they create a set with the given group of elements. They orientate themselves in time; they compare the size of shapes. Graphs and diagrams are suitable means for the development of children´s skills. The article focuses on children´s activity in the elementary school.

Antonín Jančařík ⟨antonin.jancarik@pedf.cuni.cz⟩ · **Charles University in Prague**

Mají budoucí učitelé matematiky umět počítat?
Představení několika početních algoritmů, kterými je obohacována příprava budoucích učitelů matematiky, aneb co studenti učitelství neznají ze základní a střední školy.

Mária Jurečková ⟨maria.jureckova@ku.sk⟩ · **Catholic University in Ružomberok**

Educational research of the mathematical competence
The aim of this paper is to identify the development of the level of mathematical knowledge in secondary schools. We focus on the comparison of results of nation-wide tests in mathematics carried out by the National Institute for Certified Educational Measurements during the last six years. These tests measured the pupils' skills of applying basic mathematical principles and procedures in both the mathematical context and real life. Special attention is given to the analysis of the pupils' knowledge in different thematic units, gender analysis and comparison of pupils' achievements according to the founder of the school.

Petra Konečná ⟨petra.konecna@osu.cz⟩ · **Jakub Poruba** ⟨j.poruba@centrum.cz⟩ · **University of Ostrava**

The state of pre-gradual preparation of Math teachers at the time of the end of existence of branches of study
According to the amendment of Higher Education Act, prolonging of any contemporary accredited branches of study will not be possible. By the day of the end of the accreditation validity, contemporary form and structure of branches of study including branches focused on teacher training preparation will come to an end. Thus, the aim of this contribution is to summarise the state of pre-gradual preparation of Math teachers throughout Czech universities, to compare contemporary situation with new

demands of recognizing authority at the area of regulated profession – teacher, and to introduce possible concept of entire teacher training preparation on bachelor's and postgraduate master's level.

Péter Körtesi ⟨pkortesi@gmail.om⟩ · **University of Miskolc**

Hand held technology in Mathematics

We will analyse the possible use of hand held technology to study Mathematics, including the smart phones versions of some softvare. We will give examples for the advantages and disadvantages of some sofvare.

Jana Krajčiová ⟨jana.krajciova.kosice@gmail.com⟩ · **Gymnázium Alejová, Košice**

Rapsodické skúsenosti učiteľky matematiky

Aké sú rozdiely na vyučovacích hodinách v klasických triedach, v triedach s rozšírenými hodinami matematiky či v triedach s intelektovo nadanými žiakmi? Prečo je niektorá trieda aktívna, iná pasívna? Ako primäť žiakov k aktivite? Čo trápi učiteľov? Čo trápi žiakov? Veľa otázok, ťažké hľadanie odpovedí.

Ingrid Nagyová ⟨ingrid.nagyova@osu.cz⟩ · **University of Ostrava**

Didactics of mathematics versus didactics of informatics

Mathematics and informatics have evolved together over the centuries and as separate sciences have established themselves for the last hundred years. The contribution deals with the analogies which follow from the joint development of the two branches and which reflect afterwards in their didactics. It focuses on common principles of teaching mathematics and informatics in the modern period of digital technologies.

Martin Papčo ⟨papco@ruzomberok.sk⟩ · **Catholic University in Ružomberok**

Why should we use indicator functions?

An indicator function or a characteristic function is a function defined on a universe that indicates membership of an element in some subset of the universe. Its simplicity seemingly suggests its uselessness. But it is quite the other way around. An indicator function represents effective mathematical tool in several fields of study. For instance, it is significantly favourite in combinatorics

and probability theory. But it provides simplified proofs of various relationships and properties in other areas, too. I intend to present several situations in which the notion enable us to do mathematics easier and not so cumbersome.

Edita Partová ⟨`partova@fedu.uniba.sk`⟩ · **Comenius University in Bratislava**

Dilema medzi presnosťou a zrozumiteľnosťou vo vyučovaní matematiky
Teórie vyučovania matematiky sa zhodujú v tom, že abstraktné pojmy je potrebné modelovať. Prvý model spravidla neukazuje všetky detaily pojmu, upresňuje sa postupne v súlade s kognitívnym vývinom žiaka. V prednáške uvedieme príklady zjednoduše-ných modelov a opisov abstraktných pojmov na začiatku školskej dochádzky, ktoré postupne prehlbujeme a rozširujeme. Modely a charakteristiky musia byť zrozumiteľné a pravdivé aj keď nie úplné.

Adam Płocki ⟨`adplocki@up.krakow.pl`⟩ · **State Higher Vocational School**

Probability around us, or what, how and why is mathematized
A coin or dice toss, as well as drawing balls out of urns are simulation schemes (and also mathematical models) of many real life phenomena. The paper concerns the mathematization phase of maths use process and the role of dice and coins in this phase organisation.

Jiří Přibyl ⟨`jiri.pribyl@ujep.cz`⟩ · **Petr Eisenmann** · **Jan Evangelista Purkyně University in Ústí nad Labem**

Heuristic Strategies in Graph Theory Problem Solving
In this paper we describe some examples of graph theory problem solving via heuristic strategies. These problems are solved in the graph theory lectures in bachelor's degree program in the study field Information Systems at the Jan Evangelista Purkyne University. The students used following nontrivial heuristic strategies spontaneously: Analogy, Omitting a condition, Decomposition into simpler cases and Using of invariant.

Luljeta Sadiku ⟨luljeta.sadiku@yahoo.com⟩ · **International Balkan University**
Murat Sadiku ⟨m.sadiku@seeu.edu.mk⟩ · **Izet Zeqiri** ⟨i.zeqiri@seeu.edu.mk⟩ · **South East European University**

An Econometric Model for financing Higher Education in the Republic of Macedonia

The higher education system is considered to be one of the main pillars of a country's economic development. The application of a qualitative model of financing the higher education and investment in human capital should be priority of the government. Thus, the aim of this paper is to analyze the model of financing the higher education in the Republic of Macedonia. By means of an econometric model will be estimated the effects of the higher education financing of some universities and its impact on the economic development of the country. The results indicate that the current approach of financing the higher education gives a small positive effect on the economic development of the country.

Murat Sadiku ⟨m.sadiku@seeu.edu.mk⟩ · **Nimete Berisha** · **South East European University**

A Method on Accelerating Convergence of Trigonometric Series

In the paper we use an operator of generalised difference, defined earlier, linear on a set of sequences, in order to establish a limit case Euler transform for trigonometric series. By using a property of the operator, we establish Euler type transforms for alternating series. These transforms accelerate the convergence of trigonometric series under the same conditions as the transforms of non-alternating series.

Katarína Sátorová ⟨mojsovak@gmail.com⟩ · **Daniela Guffová** · **Matej Bel University in Banská Bystrica**

Argumentation and reasoning of prospective teachers

The purpose of this paper is to investigate the reasoning and argumentation of prospective teachers in basic mathematical analysis course. Examples of analysis of exam tasks are presented and followed by analysis of students' argumentation and reasoning. Theoretical background for the analyses is based on the integration of the Toulmin structural description of argument, the Peirce notion of sign and the Lithner classification of reasoning complemented by the Bergqvist classification of the tasks.

Qianjun Tang ⟨57225351@qq.com⟩ · Jitka Laitochová · Palacký University in Olomouc

How to Integrate ICT into Mathematics Teaching?

The terminology integration or fusion has been in our education field since beginning the new millennium. Actually, some of us don't comprehensively understand the core of the terminology and do not know how to practice the new way of teaching. The author thinks there are four parts or procedures in the teaching activity, namely, curriculum's goals, the content, the teaching process, and the assessment. Based the reasons above, the process of integration of ICT and mathematics course should run throughout the whole procedure of designing and developing mathematics course, which includes the whole integration of ICT and mathematics curriculum criteria, contents, implementation, and assessment. We can explore the strategies of integration from the four aspects above. What is more, during the integration, we should focus on cultivating competency and transmitting knowledge as well, jointing the inquiry learning and accepting learning, unifying students' dominant role and teachers' guidance role, and combining the instructional technology and art.

Zuzana Václavíková ⟨zuzana.vaclavikova@osu.cz⟩ · University of Ostrava

LARPing the Math

The implementation of Live Action Role Playing Games has increased dramatically in recent years, especially in the field of science subjects. However, mathematics has a special position from this point of view because it works more like a tool for "describing the worldänd is often taught without an application context. In LARPs focused on natural sciences, there is a need to use mathematical knowledge occasionally, but LARPs that are geared directly to mathematical areas are rare. This article aims to introduce our experience with LARPs that are geared directly to specific mathematical areas.

Mateusz Woronowicz ⟨mworonowicz@math.uwb.edu.pl⟩ · Ryszard R. Andruszkiewicz · University of Białystok

On the square subgroup of an abelian group

An abelian group $(A, +, 0)$ is called a nil group if on A there does not exist any nonzero ring multiplication. Nil groups were studied for a long time by many authors and there are several generalizations. In the talk we will present one of them. Namely, we will show the latest results concerning the square subgroup of an abelian group and we will explain the historical background of this research. Given an abelian group A, the square subgroup $S(A)$ of A can be understood as the subgroup of A generated

by squares of all possible rings defined on A. There is a natural question whether the quotient group of A modulo $S(A)$ is a nil group. For torsion groups the (positive) answer is well-known. However, for mixed and torsion-free groups the above-mentioned question remained unanswered for 35 years. The (negative) answers were given by A. Najafizadeh, R. R. Andruszkiewicz and M. Woronowicz in 2015 and 2016.

Bożena Woźna-Szcześniak ⟨bwozna@gmail.com⟩ · **Ireneusz Szcześniak** · **Jan Długosz University in Czestochowa**

Real-Time Commitment Logic interpreted over Duration Communication Interpreted System

In this paper, we develop a system of real-time logic TCTLC, an extension of the TCTL logic with conditional commitment and their fulfillment modalities, which is interpreted over the Duration Communication Interpreted System (DCIS), i.e., the communication interpreted system, where transitions have integer durations. This logic framework allows us to formally model the behaviour of agents using conditional commitments and real-time constraints in order to permit reasoning about qualitative and quantitative requirements.

Izet Zeqiri ⟨i.zeqiri@seeu.edu.mk⟩ · **Murat Sadiku** ⟨m.sadiku@seeu.edu.mk⟩ · **South East European University**
Luljeta Sadiku ⟨luljeta.sadiku@yahoo.com⟩ · **International Balkan University**

A Mathematical Model for financing Higher Education

In this paper will be explained an optimal way of financing higher education using a mathematical model. Through that model will be revealed which factors motivate employees in universities to be as productive as possible and what impact universities give in the economic development.

Pavol Zlatoš ⟨Pavol.Zlatos@fmph.uniba.sk⟩ · **Comenius University in Bratislava**

Hilbert's Program and Gödel's Incompleteness Theorems

We will briefly discuss the crisis in the foundations of mathematics at the turn of the 19th and 20th century caused by the discovery of paradoxes in Cantor's set theory as well as in Frege's Begriffsschrift, and the reactions they caused. Then we will explain the reasons which led David Hilbert to the formulation of his program and outline the main ideas of his attempt at the proof of the consistency of the foundations of mathematics. Finally, we will sketch the essentials of Gödel's Incompleteness Theorems and their relation to Hilbert's Program. In the course we will try to refute some popular myths about these issues.